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The Effect of Verbalizer, Spatial and Object Visualizer Cognitive Style to Students' Algebraic Thinking: Insights from Empirical Study Focusing on Middle School Students

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ABSTRACT

Everyone has a preferred approach and different habits in organizing and representing the information they received. It is known as cognitive style. The theory of verbalizer and visualizer cognitive style developed into three dimensions; verbalizer, object visualizer and spatial visualizer. Algebraic thinking consists of four components, namely generalized arithmetic (GA), functional thinking (FT), modeling languages (M), and algebraic proof (AP). Previous research has shown that spatial visualizers have a more significant relationship with numbers sense and algebraic reasoning compared to verbalizers and object visualizers. However, there is no empirical research that shows the effect of three-dimensional cognitive style on the 4 components of algebra, especially for students in junior high school. The subjects of this study were 149 8th grade students in Gresik and Sidoarjo regencies, Indonesia. This study uses Structural Equation Modeling (SEM) data analysis with AMOS 24. The instrument used is an adaptation of the Object- Spatial Imagery and Verbal Questionnaire (OSIVQ) Blazhenkova & Kozhevnikov (2009) and an algebraic thinking test instrument. The results of this study indicate that three-dimensional cognitive style affects the components of algebraic thinking, especially in mathematical modeling (M). In addition, functional thinking (FT) has the greatest contribution to algebraic thinking skills compared to GA, M and AP.

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INTRODUCTION

Students at junior high school experience a transition period from early algebraic thinking to more advanced. Radford (2000) states that algebraic thinking is a person's attempt to represent generalizations in various ways by gestures, expressions using words, pictures or symbols consisting of letters and numbers. Kieran (1996) explains that algebraic thinking involves a way of thinking involves symbols as a tool to analyze the relationship between quantities, generalization, problem solving, modeling, justification, proof and conjecturing. In addition, algebraic thinking can also be seen as a habit of students in recognizing and expressing structures and relationships in mathematics J. J. & B. M. Kaput, 2005; Booker & Windsor, 2010 . According to Kaput(J. J. Kaput et al., 2008) There are 4 (four) main branches in algebraic thinking, namely generalized arithmetic (GA), functional thinking (FT), modeling languages(M), and algebraic proof (AP). Generalized arithmetic is defined as generalization of arithmetic operations (addition, subtraction, multiplication and division) and their properties, looking for the structure of arithmetic relationships and not focusing on the results of their calculations. The equal sign "=" and equivalence are the basis of generalized arithmetic. Functional thinking is concerned with situations in which a person tries to find a way to express systematic variations from examples. Functional thinking involves several algebraic concepts such as equations, variables, variations, and correspondences. Applications of modeling languages are used to explain generalizations. Meanwhile, algebraic proof requires students to understand the abstract nature of algebra and the concept of proof.

Cognitive style defined as a person's tendency to organize and represent the information received. Therefore, it is closely related to thinking process. Cognitive style also reflects someone's personality, including students. One of the most commonly used at educational studies is the visual-verbal cognitive styles by Paivio (1971) or Richardson (1977). A general idea of visual and verbal cognitive style was there two different visual and verbal processing systems. Blazhenkova & Kozhevnikov (2008) proposed a new theoretical model that distinguishes between object and special imagery which known as three-dimensional model. In order to develop spatial transformation abilities, spatial visualizers tend not to maintain a lot of pictorial details in their images. Meanwhile, object visualizers, in contrast, develop many pictorial details. Self-report questionnaire then developed by Blazhenkova & Kozhevnikov (2009) which is known as the Object-Spatial Imagery and Verbal (OSIVQ) framework. Since proposed at 2009 the instruments were translated into different languages, for instance Kawahara & Matsuoka (2012) who translated into Japanese and the participants were undergraduate students. (Blazhenkova, 2016) examined the vividness of object and spatial visualization with undergraduate students. Elhan Selcuk H & Mark Lavenia (2017) used this framework to investigate Cognitive styles of High School Students (17-18 years old). Until this research was being conducted, none of the Indonesian researcher developed and used the framework especially for Junior High School Students (14-15 years old).

A few numbers of recent studies (e.g., Anderson, Casey, Thompson, Burrage, Pezaris & Kosslyn 2008; Kozhevnikov et al. 2005; Chrysostomou et al., 2013) investigating the effects of three-dimensional cognitive style on their mathematical performance. Their results indicated that spatial visualizer related to success in mathematics than object visualizer. According to these findings, we need to investigate further in which cognitive styles influence students' mathematical performance in many ranges of concepts. This study supports the view that there are three dimensional cognitive styles, verbalizer and two types of verbalizers which is spatial and object visualizer. Since students are the main component in the learning process then we curious to know whether junior high school students' cognitive style affect their algebraic thinking. Based on the background, the purpose of this study was to determine the effect of cognitive style (verbalizer, spatial and object visualizer) on students' algebraic thinking and

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to determine which components of algebraic thinking have a significant impact on students' algebraic thinking.

Algebraic Thinking

Algebraic thinking has attracted the attention of researchers from early years and at various levels of education. Different views have been expressed to defines algebraic thinking Kieran 1996;Radford, 2000; J. J. Kaput et al., 2008, Booker & Windsor 2010. However, there is consensus that a person to be said thinking algebraically if he/she can action on generalizations and express it using conventional symbols. Moreover, it was described that algebraic thinking consists of 4 (four) branches, namely: 1) Generalized arithmetic, 2) Functional thinking, 3) Modeling languages, and 4) Algebraic Proof ((J. J. Kaput et al., 2008)

Building generalized arithmetic is the most important in algebra. Included in this component are the generalization of arithmetic operations and their properties as well as looking for relationships between structures in arithmetic. In addition, generalized arithmetic also includes generalizations about the properties of certain numbers and expressions that express calculation strategies. The "equals" sign and the equation are fundamental to algebraic generalizations. Blanton and Kaput (2005) stated that the tasks that can be given to generalize algebra are: 1) exploring the properties of numbers 2) exploring the properties of number operations, 3) exploring equations that express the relationship between quantities, 4) treating numbers algebraically, 5) complete the math sentences. Functional thinking involves generalizing on functions. Smith (2008) states functional thinking as representational thinking that is focused on the relationship between two or more different quantities. Blanton and Kaput (2005) state that tasks that can be given for functional thinking are: 1) symbolizing numbers with variables or operating variables, 2) presenting data with graphs, 3) finding functional relationships, 4) predicting unknown numbers based on data, 5) identify and explain arithmetic sequences, 6) identify and explain geometric sequences. The third component is to use modeling languages to explain generalizations. According to Kaput (2008) there are 3 (three) models, namely 1) a model that states a certain number or quantity. This model is usually a limiting statement, for example in the form of an equation that uses a variable as an unknown number. 2) Generalization of the modeled situation, usually using an algebraic form with one or more variables that can represent a function. 3) Generalization of the completion of a model of the situation for example math story problems. Algebraic thinking related to proof can be seen in 3 (three) forms, namely 1) using generalizations to build other generalizations, 2) generalizing mathematical processes or formulas, 3) testing conjectures, justifying and proving. It is undeniable that the proving process is a complex matter for students (Healy & Hoyles, 2000).

The results of Pitta-Pantazi et al., (2020) on 50 junior high school students at grades 8th and 9th show that functional thinking is the basis for other components of algebraic thinking. If students have functional thinking skills, then they can only move to generalized arithmetic. Functional thinking and generalized arithmetic are the stepping stones for a modeling language which can then be used as a basis for students to construct algebraic proofs.

Cognitive Style

Witkin defines cognitive style as a process, individual differences in accepting, thinking, solving problems, learning, and relating to others(Witkin et al., 1977). Cognitive style is stable over time(Witkin et al., 1977), (Ausburn & Ausburn, 1978). Under normal conditions, we can predict that a person with a certain cognitive style today will have the same cognitive style the next day, the next month or even the next year. Another opinion by Riding & Rayner (1998) in Sternberg & Sternberg (2012) states that cognitive style is an approach that is preferred by a person and his habits of organizing and representing the information he receives. This opinion states explicitly that cognitive style is related to the way a person represents information even though the information representation itself is a part of information processing.

Many researchers come to a consensus that categorizing students into visualizers and verbalizers is too general (Kozhevnikov et al. 2002; Kozhevnikov, Kosslyn, and Shepard 2005; Blazenkhova, 2016). Kozhevnikov, et al (2005) presents cognitive style in three dimensions, namely verbalizer cognitive style, object visualizer and spatial visualizer. The separation of visualizer cognitive style into object visualizers and spatial visualizers were also supported by several experts such as Blazhenkova & Kozhevnikov (2009)and Haciomeroglu & LaVenia (2017). Koć-Januchta, Hoffle, et al. (2017) explain that there is a difference between visualizer and verbalizer in looking at pictures and texts attentively during learning. By investigating using eye tracking behavior, it is found that the visualizer spends more time examining images than the verbalizer. Meanwhile, verbalizer spends more time checking text or words. Verbalizers tend to enter information and areas of irrelevant images faster than visualizers. In addition, the results of the analysis also show that there are differences in learning outcomes between the visualizer and verbalizer. The visualizer group scored better on the comprehensive test than the verbalizer group. Object visualizer draws an in detail, obstructing effective spatial transformation and good performance in mathematical and spatial task. Spatial visualizer tends to use imagery to explain spatial relationship between objects so that facilitate spatial transformation efficiently and better performance in mathematical and spatial task (Haciomeroglu & LaVenia, 2017) .

Blazhenkova & Kozhevnikov (2009) who developed the Object-Spatial Imagery and Verbal Questionnaire (OSIVQ), a questionnaire that aims to assess differences in individual cognitive styles based on their perception of words, object images and spatial images. They showed that OSIVQ demonstrated acceptable validity on construct, criterion and ecological as well as internal reliability. Studied by Haciomeroglu & LaVenia (2017) with a subject of 348 students in high school shows that the spatial visualizer cognitive style has a significant correlation with the object visualizer cognitive style, spatial visualization ability and verbal-logical reasoning ability. While the cognitive style of object visualizer does not have a significant correlation with any measure of cognitive ability. Spatial image style is not correlated with object image style and negatively correlated with verbal style. The object image style was not significantly correlated with any measure of cognitive ability, while the spatial image style was significantly correlated with the object image ability, spatial visualization ability, and verbal-logical reasoning ability. Lastly, spatial imagery style and verbal-logical reasoning ability significantly predict students' preference for efficient visual methods. The results support the cognitive style model, in which visualizers are characterized as two distinct groups who process visual-spatial information and graphic tasks in different ways. Research conducted by Chrysostomou et al. (2013) on 83 prospective teachers shows that there is a relationship between cognitive style (verbalizer, object visualizer and spatial visualizer) with sense of numbers (number sense) and algebraic reasoning (algebraic reasoning). Spatial visualizer has a more significant relationship with sense of numbers and algebraic reasoning compared to verbalizer, object visualizer. These results also imply that individuals who have a high spatial visualizer cognitive style have more flexible and conceptual strategies in completing tasks related to number sense and algebraic reasoning.

METHODS

Participants

This research is a type of quantitative research that is deductive in nature. Data were grouped, concrete, relatively fixed, and measurable. We answered research questions by formulating hypotheses

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and testing them. The subjects in this study were 149 junior high school students at eighth grade, 13-14 years old, consisting of 39 males and 110 females. The students came from 2 public schools in Gresik Regency and 1 private school in Sidoarjo regency, Indonesia. The schools are SMP Negeri 1 Gresik, SMP Negeri 2 Gresik, and SMP Bilingual Terpadu Sidoarjo. Subject selection was based on school permit and students' willingness to join the research. It was conducted in the even semester of the 2019/2020 academic year and the odd semester 2020/2021 during the Covid-19 pandemic. Therefore, data collection is carried out online or offline in accordance with government and school policies.

Instruments and Procedure

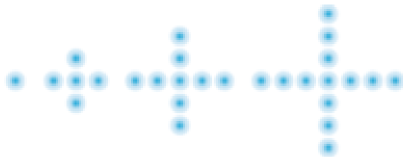
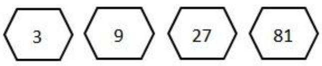
The variables in this study are divided into 2 (two) namely independent variables and dependent variables. The independent variable is cognitive style, verbalizer, object and spatial visualizer. The dependent variable is algebraic ability which consists of generalized arithmetic, functional thinking, modeling language and algebraic proof. To obtain research data, we used the questionnaire method to collect students' cognitive style and a test method for algebraic thinking.

Algebraic Thinking Test

The algebraic thinking test, some of it is presented in Table. 1, aimed to assess students algebraic thinking. We developed the task in three ways. As the first step, we investigated the indicators of each component of algebraic thinking according to (J. J. Kaput et al., 2008) which is generalized arithmetic abilities (GA), functional thinking (FT), modeling language (M) and algebraic proof (AP). We then selected the kind of tasks used in previous studies, adjusted these in Indonesian context and curriculum. The algebraic thinking instrument consists of 24 questions, consists of 9 items for generalized arithmetic, 6 items for functional thinking, 6 items for modeling language and 3 algebraic proofs. In order to get construct validity, we consult with two validators, a mathematics education lecturer at Universitas Muhammadiyah Gresik, Indonesia and a mathematics teacher at SMP Negeri 2 Gresik, Indonesia. Both of those validators experienced in teaching more than 15 years. Every task is an open question, first participants were asked to read the instruments after that they give their answer and explain their reasoning toward it. We give remark from 1 to 5 according to their answer, in which 1 if the answers in incorrect to 5 if the answer is correct and complete or using generalization in reasoning. There was no time limit was required for answering the algebraic thinking test. The table below show the indicators of algebraic thinking test.

Table 1. Some Examples of Algebraic Thinking Test

No	Indicators	Example of test
A	*Generalized arithmetics*	
1	Exploring relation of integers	Without counting, Determine whether the result of 3428952671+2467151458 is an even or odd Explain your answer
2	Exploring the properties of number operations	Without counting the sum of each 2 numbers, determine the result of $(-53)+(-52)+(-51)+\cdots+(-1)+0+1+\cdots+48+49+50$
3	Exploring equations that express the relationship between quantities	Replace the box below with a number so that it produces a true statement $72+(-5)=\square+8$

4	Treating numbers algebraically	Which of the following two exponents results in the larger number 10^{11} atau 11^{10}
5	Complete the math sentence	If  express a number and  +  = 24 Find the number of  +  = 6, Explain your answer
B	*Functional Thinking*	
1	Symbolizing numbers with variables or operating variables	Ahmad plays with a toy car and moves the car so that at the 5th minute the length of the track the car travels is 12 meters, at the 6th minute it is 14 meters and at the 10th minute it is 22 meters. Present data on pairs of times and the length of the path traversed by the toy car using the Cartesian coordinate plane
2	Presenting data with graphs	Determine the length of the path traveled by Ahmad's toy car in the 25th minute. Explain your answer.
3	Finding functional relationships	Dani arranges the balls into 4 configurations as shown below!  If he continues to arrange the balls, how many balls make up the 10th configuration? Explain your answer.
4	Predicting unknown numbers based on data	Given a graph shows the costs incurred for purchasing admission tickets to a safari park for adults. How much does it cost to buy 13 tickets for adults? Explain your answer.
5	Identify and explain rithmetic sequences	In class decorating competition, students of VIII C SMP Cendekia arrange pieces of paper in the hexagon shapes in which numbers arranged according to certain rules as shown bellow.  Find the number at 18th order. Explain your answer.
6	identify and explain geometric sequences	VIII D students arrange pieces of paper with different arrangement of numbers as shown below.  Find the number at 6th order. Explain your answer

The Cognitive Style Questionnaire

Questionnaire method is used to obtain the three-dimensional cognitive style of students. We developed Indonesian version of Object-Spatial Imagery and Verbal Questionnaire (IDN-OSIVQ). First,

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we translated 45 items OSIVQ by Blazhenkova & Kozhevnikov (2009)and rate scales of the OSIVQ into bahasa. After the translation, we consulted the English interpreter. Since the IDN-OSIVQ used for Junior High School Students, then we consulted the instruments to the psychologist to get advices whether the terms used appropriate with their age, knowledge and experience. Table 2 shows some of IDN-OSIVQ compare to OSIVQ. The questionnaires also used a- five-point scale from 5 = strongly agree to 1 = strongly disagree or vice versa for negative statements. For each type of cognitive style there are 15 (fifteen) questions. The cognitive style questionnaires (IDN-OSIVQ) and test instruments are distributed offline or online depend on school policy. To examine the factor structure and internal consistency reliability of the IDN-OSIVQ, the questionnaire was administrated to the participants. We informed about the purpose of the research and assured to the students that it was not their mathematics remark then we asked them to read every item in the I-OSIVQ and respond by choosing one of the points on the scale. No time limit was required for the completion of the questionnaire.

Table 2. Some Statements Of IDN-OSIVQ

No	OSIVQ	IDN-OSIVQ
	Verbalizer	
2	I have difficulty expressing myself in writing	Saya kurang mampu mengungkapkan perasaan saya melalui tulisan.
4	My verbal abilities would make a career in language arts relatively easy for me	Saya merasa kemampuan verbal saya dapat menunjang pilihan saya untuk menekuni seni bahasa
	Spatial Visualizer	
5	Architecture interests me more than painting	Saya lebih tertarik pada arsitektur daripada melukis
7	I prefer schematic diagrams and sketches when reading a textbook instead of colorful and pictorial illustrations	Saat membaca buku, saya lebih suka membuat (mencoret-coret) diagram dan sketsa, daripada membayangkan gambar yang penuh warna.
	Object Visualizer	
6	My images are very colourful and bright	Saat menggambar saya suka menggunakan banyak warna yang cerah
11	When reading fiction, I usually form a clear and detailed mental picture of a scene or room that has been described	Saat membaca buku cerita, saya bisa membayangkan kejadian atau ruangan yang diceritakan dalam cerita itu secara mendetail dan jelas

Validity and Reliability of the Instruments

We used the convergent validity test to measure the validity of each item. An item is said to be valid if the loading factor value is ≥ 0.7. Based on the results of the analysis of the convergent validity test, it was obtained that for the verbalizer cognitive style instrument, out of 15 items there were 11 (eleven) valid items, for the spatial visualizer cognitive style there were 11 valid items while for the object cognitive style there are 12 valid items. As for the algebraic thinking ability instrument, based on the convergent validity test displays on table 3, it was found that all indicators of the GA, FT, M and AP variables had met convergent validity with a loading value ≥ 0.7. Reliability test shows the extent to which a measuring instrument can provide the same results when repeated measurements are made on the same object. The minimum reliability value of the dimensions forming t he latent variable that can be accepted is ≥ 0.70. The following table presents the results of the reliability test for each instrument.

Table 3. Reliability Test Result

Factor	Construct Reliability Value
Verbalizer (V)	0,9492
Visualizer Spatial (VS)	0,9576
Visualizer Object (VO)	0,9708
Generalized Arithmatics (GA)	0,9579
Functional Thinking (FT)	0,9439
Modelling Mathematics (M)	0,9295
Algebraic Proof (AP)	0,9686

FINDINGS AND DISCUSSION

Findings

In this part, we'll explain our research findings supported by the data

The Effect of Verbalizer on Students' Algebraic Thinking

The first aim of the study was to determine the effect of cognitive style (verbalizer, spatial and object visualizer) on students' algebraic thinking (GA, FT, M and AP). First we carried out the test by analyzing the CR and P value from the results of SEM data processing. After that we compared with the required statistical limits ≥ 1.96 for the CR value and ≤ 0.05 for the P value.

Table 4. Parameter Estimation of The Effect of Verbalizer (V) On The Components of Algebraic Thinking

			Estimate	S.E.	C.R.	P
GA	<---	V	,047	,126	,372	,710
FT	<---	V	,051	,118	,435	,664
M	<---	V	,198	,099	1,993	,046
AP	<---	V	-,067	,142	-,468	,640

Table 4 reveals that the parameter estimation of the effect of variable V on the components of algebraic thinking shows a value that varies from CR with a range of -, 468 to 1.9993. GA (CR < 1.96 and P> 0.05), FT (CR < 1.96 and P > 0.05), M (CR > 1.96 and P 0.05) and AP (CR < 1 .96 and P 0.05). Based on this value, the variable V only affects M. This finding also supported by SEM on Fig. 4.

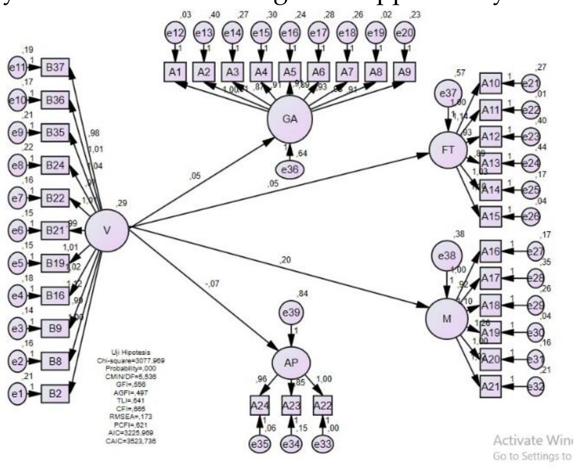

Figure 1. Structural Analysis of The Verbalizer (V) and the Components of Algebraic Thinking Model

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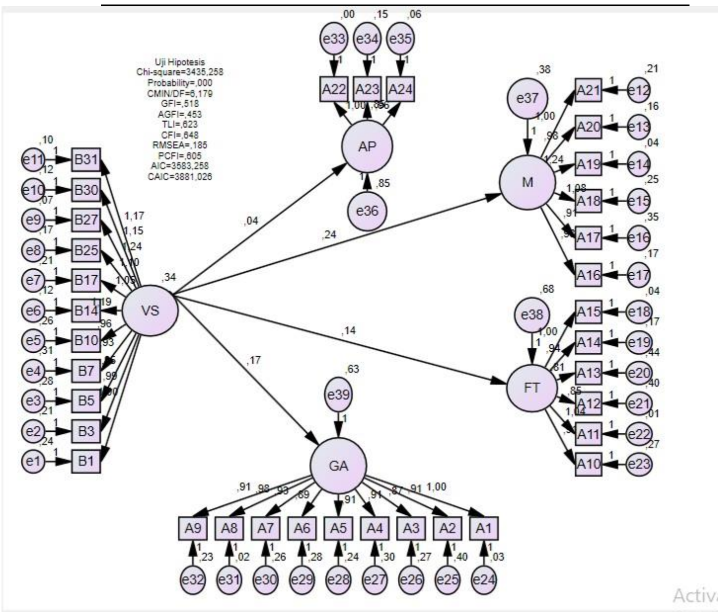
The Effect of Spatial Visualizer on Students' Algebraic Thinking

Table 3 presents that the estimation parameter of the influence of the VS variable on the component of algebraic thinking shows a value that varies from CR with a range of .297 to 2.648. GA (CR < 1.96 and P> 0.05), FT (CR < 1.96 and P > 0.05), M (CR > 1.96 and P 0.05) and AP (CR < 1 .96 and P > 0.05). Based on this value, the VS variable only affects M. This finding also supported by SEM on Fig. 5.

Table 5. Parameter Estimation of The Effect of Spatial Visualizer (VS) on The Components of Algebraic Thinking

			Estimate	S.E.	C.R.	P
GA	<---	VS	,170	,115	1,488	,137
FT	<---	VS	,144	,118	1,223	,221
M	<---	VS	,245	,092	2,648	,008
AP	<---	VS	,038	,130	,297	,767



Figures 2. Structural Analysis of The Spatial Visualizer (VS) and the Components of Algebraic Thinking Model

The Effect of Object Visualizer on Students' Algebraic Thinking

Table 6 presents that the estimation parameter of the influence of the Vo variable on the component of algebraic thinking shows a value that varies from CR with a range of -.687 to 6.930. GA (CR > 1.96 and P> 0.05), FT (CR > 1.96 and P > 0.05), M (CR > 1.96 and P 0.05) and AP (CR < 1 .96 and P > 0.05). This implies that the influence of the variable Vo on the GA, FT and M components is statistically significant. Based on this value, the Vo variable not affects AP. This finding supported by SEM on Fig. 3.

TABLE 6. Parameter Estimation of The Effect of Object Visualizer (Vo) On the Components of Algebraic Thinking

			Estimate	**S.E.**	**C.R.**	**P**
GA	<---	VO	,520	,075	6,930	***
FT	<---	VO	,480	,076	6,350	***
M	<---	VO	,483	,061	7,868	***
AP	<---	VO	-,064	,093	-,687	,492

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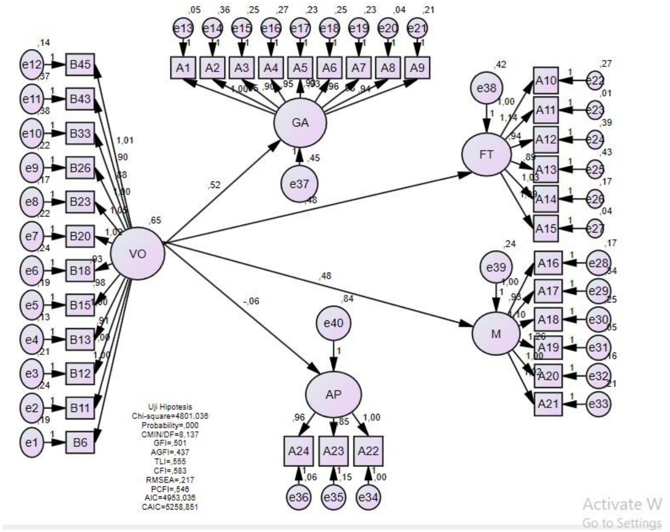

Figures 3. Structural Analysis of The Object Visualizer (VO) and the Components of Algebraic Thinking Model

General Model of Cognitive Style Effect with Algebraic Thinking Component

The estimated parameters for testing the effect of cognitive style variables on learning abilities in table 5 show a CR value of 3.137 with a probability value of 0.002. This value has met the requirements for H1 acceptance, which is a probability less than 0.05 and a CR above 1.96. So it can be concluded that there is an influence of cognitive style on algebraic thinking ability.

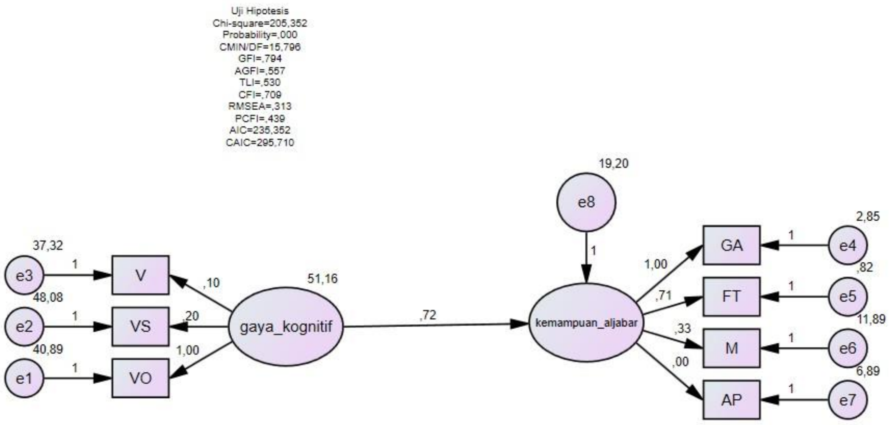

Table 6. Parameter Estimation of GA, FT, M, and AP

			Estimate	S.E.	C.R.	P	Label
GA	<---	Algebraic Thinking	1,000				
FT	<---	Algebraic Thinking	,706	,024	29,657	***	par_3
M	<---	Algebraic Thinking	,327	,042	7,703	***	par_4
AP	<---	Algebraic Thinking	,001	,032	,018	,985	par_5

Based on the significance value of the estimated Regression Weights parameter, GA and FT provide a significant relationship to Algebraic Thinking except for M and AP.

Table 7. Factor score weights

	AP	M	FT	GA
Algebraic Thinking	,000	,028	,859	,352

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From Table 7. We know the strength and direction of the relationship between AP, M, FT and GA and Algebraic Thinking. Among those components, Functional Thinking has positive and the most significance influence with 0,859 score. On the other side, there is no influence of Algebraic Proof toward algebraic thinking.

Discussion

The results of this study indicate that three-dimensional cognitive style affects the components of algebraic thinking, especially in mathematical modeling (M). Prior research has consistently demonstrated that individuals' cognitive styles, such as verbalizer and visualizer tendencies, influence how they process and understand mathematical information (Blazhenkova & Kozhevnikov, 2009;Riding & Rayner, 1998).Individuals with a spatial visualizer cognitive style tend to excel in tasks involving spatial reasoning, visualization, and geometric problem-solving, which are closely related to mathematical modeling and certain aspects of algebraic thinking (Hegarty & Kozhevnikov, 1999). Moreover, theoretical frameworks such as the Dual Coding Theory(Paivio, 1971b) suggest that individuals encode and represent information in both verbal and visual formats. This theory implies that cognitive styles, which influence the preference for verbal or visual processing, can impact how individuals approach mathematical tasks and problem-solving strategies.

Additionally, research in mathematics education has explored the relationship between cognitive styles and mathematical performance, highlighting the importance of considering individual differences in instructional design and pedagogical strategies (Ainsworth 2006; Goldin & Kaput 1996) how cognitive styles influence algebraic thinking can inform the development of tailored interventions and instructional approaches to accommodate diverse learning preferences and enhance mathematical learning outcomes.

The finding that functional thinking has a positive and significant influence on algebraic thinking aligns with prior research and theoretical frameworks in mathematics education. Functional thinking involves understanding relationships between variables and analyzing patterns and functions, which are foundational to algebraic reasoning. Numerous studies have highlighted the importance of functional thinking in developing algebraic skills, emphasizing its role in fostering abstraction, generalization, and problem-solving abilities essential for success in algebra (e.g., J. J. & B. M. Kaput 2005; National Council of Teachers of Mathematics, 2000). However, the observation that there is no influence of algebraic proof on algebraic thinking may seem contrary to traditional perspectives in mathematics education, where algebraic proof is often considered a fundamental aspect of algebraic reasoning. Prior research and theoretical frameworks emphasize the importance of proof in developing students' understanding of mathematical concepts, including algebraic structures and relationships (e.g., Harel, 2008; Schoenfeld, 1994).

This finding could prompt further investigation into the effectiveness of different instructional approaches in teaching algebraic proof and its impact on students' algebraic thinking skills. It may also suggest that while algebraic proof is essential for advanced mathematical reasoning, its direct influence on basic algebraic thinking among junior high school students may be limited compared to other components, such as functional thinking. While the finding regarding functional thinking aligns well with prior research and theories emphasizing its significance in developing algebraic thinking skills, the observation regarding algebraic proof may warrant further exploration and could potentially challenge traditional perspectives on the role of proof in early algebraic education.

CONCLUSION

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This study shows that in general, the cognitive style of verbalizer, spatial visualizer and object visualizer affect students' algebraic thinking skills, especially on the component of Functional Thinking (FT). The cognitive style of object visualizer contributes more to several components of algebraic thinking skills, namely GA, FT, and M. In addition, the results of the analysis also show that students' algebraic thinking skills in junior high school consist of 4 branches, namely GA, FT, M and AP. All types of three-dimensional cognitive styles have a contribution especially to the mathematical modeling of students. The students' algebraic abilities based on their level are determined by functional thinking ability, arithmetic generalization ability, and mathematical modeling. Algebraic proof empirically does not show its effect on the ability to think algebraically.

Based on the results of this study, we provide several suggestions; the first is that teachers and developers of mathematics curriculum in Indonesia should consider the hierarchy of the 4 components of algebraic thinking as the results of this study. Functional thinking has the biggest contribution in developing students' algebraic abilities. Thus, the alternative presentation that can be done is to study functional thinking first before generalizing arithmetic. The generalization process does not seem to be an easy thing for junior high school students.

The results of this study also show that empirically, the cognitive style of the object visualizer contributes more to algebraic thinking skills. In one class, maybe the number of students with this cognitive style is not too many. Therefore, teachers should provide variations in the presentation of teaching materials and media used so that students with other cognitive styles can gain the best experience and learning outcomes, especially in algebraic material. The conclusion should answer the objectives of the research and research discoveries. The concluding remark should not contain only the repetition of the results and discussions or abstract. You should also suggest future research and point out those that are underway.

ACKNOWLEDGMENTS:

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CONFLICTS OF INTEREST

We declare that they have no conflicts of interest regarding the publication of this article

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