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DESIGNING A HYPOTHETICAL LEARNING TRAJECTORY USING THE *PINEHALA* CUSTOMARY SANCTION CONTEXT TO FOSTER STUDENTS' UNDERSTANDING OF ALGEBRAIC FACTORIZATION

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A B S T R A C T

Mathematics instruction in digital classrooms has not fully utilized digital technologies and is often delivered abstractly without meaningful real-world contexts, particularly in teaching algebraic factorization, a topic widely perceived as difficult by students. Therefore, this study aimed to design a *Hypothetical Learning Trajectory* (HLT) for algebraic factorization by employing the Pinehala customary sanction of the Tolaki Mekongga community as a meaningful starting point within the framework of *Realistic Mathematics Education* (RME). This qualitative study adopted *design research*, limited to the preliminary design phase, focusing on developing and validating the proposed HLT. Data were collected through classroom observations and *Focus Group Discussions* (FGDs) involving mathematics education experts and practicing teachers. The findings showed that buffaloes (*Kiniku* or *Karambau*) and machetes (*Ta'awu*) can be represented as algebraic variables, whereas white cloth and water containers function as fixed components. Based on these findings, the HLT was designed using the iceberg model with four stages: situational, model-of, model-for, and formal model, leading students to construct the general form of algebraic factorization, $ax + ay = a(x + y)$. The HLT was implemented through a Canva-based digital Student Activity Worksheet consisting of six learning activities, supported by anticipated student conjectures and scaffolding strategies. The proposed HLT offers a contextual, meaningful, and culturally responsive approach to teaching algebraic factorization while promoting the preservation of local cultural heritage.

INTRODUCTION

The integration of digital technology into mathematics education has become an essential requirement in response to the continuous advancement of contemporary education (Sulistyowati & Asriati, 2024). In line with this development, a junior high school in Kolaka Regency established a digital classroom during the 2023/2024 academic year. Based on preliminary observations and interviews, the classroom was equipped with various digital learning resources to support mathematics instruction, including a smartboard, internet access, and individual tablets provided to each student for participation in digital learning activities. Despite the availability of these digital facilities, their educational potential has not yet been fully realized. Tablets are not routinely integrated into mathematics lessons but are used only for specific topics or learning activities that explicitly require digital support. Consequently, most mathematics instruction continues to rely on printed textbooks, whiteboards, and teacher-centered explanations. As a result, the potential of digital technologies to facilitate visualization, conceptual exploration, and interactive learning has not been fully exploited. This is particularly concerning, given that digital technologies play a crucial role in enhancing the effectiveness and efficiency of mathematics teaching and learning by promoting deeper conceptual understanding and greater student engagement (Clements & Sarama, 2024; Prado et al., 2023).

Another challenge lies in teachers' readiness to adapt to and effectively integrate digital technologies into classroom instruction (Wahyuni & Haryanti, 2024). Consequently, there is a need to develop an innovative Hypothetical Learning Trajectory (HLT) that optimizes the integration of digital technologies into mathematics instruction through the use of authentic contexts that are meaningful and closely connected to students' everyday experiences (Marniati et al., 2026). Within this framework, an HLT refers to a systematically designed learning trajectory that guides students from their initial understanding toward more sophisticated mathematical concepts through a sequence of carefully structured learning activities (Wangsa et al., 2025). Moreover, an HLT enables teachers to design coherent learning activities, anticipate students' strategies and responses, and prepare appropriate scaffolding to support students' conceptual development throughout the learning process (Umasugi et al., 2022). Accordingly, digital technologies serve not merely as instructional delivery tools but as an integral component of the learning trajectory, facilitating students' gradual construction of mathematical understanding (Andzin et al., 2024).

The rapid advancement of information technology and the increasing influence of popular culture have significantly transformed the transmission of cultural knowledge among younger generations. In Kolaka Regency, cultural knowledge that was once naturally passed down through families, customary leaders, and local communities has gradually diminished. Consequently, many students have limited opportunities to become familiar with the traditions and cultural values of the Tolaki Mekongga community (Sarmadan et al., 2025). One such tradition is the *Pinehala* customary sanction, a system of material compensation prescribed under Tolaki customary law that embodies the values of responsibility, justice, respect for social norms, and conflict resolution

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through deliberation (Pairin et al., 2024). The declining understanding of *Pinehala* among younger generations reflects a growing disconnect between local cultural heritage and students' school learning experiences. This situation underscores the need to preserve local culture by integrating culturally meaningful contexts into mathematics instruction.

In addition to students' limited familiarity with local cultural heritage, their mathematical proficiency remains a major concern. Mathematics education should not merely emphasize the acquisition of abstract concepts but should also enable students to develop meaningful conceptual understanding through authentic learning experiences (Wangsa et al., 2024). Nevertheless, mathematics continues to be perceived by many students as a difficult and unengaging subject, resulting in low levels of interest and motivation to learn mathematics (Suhartik et al., 2025). This concern is reflected in the results of the Programme for International Student Assessment (PISA) 2022, which indicate that Indonesian students' mathematics performance remains relatively low. Indonesia obtained an average mathematics score of 366, considerably below the OECD average of 465–475 (OECD, 2023). Furthermore, data reported by the Center for Educational Assessment under the Ministry of Primary and Secondary Education of the Republic of Indonesia show that the national average score on the Academic Competency Test (Tes Kompetensi Akademik, TKA) in mathematics was only 36.10. These findings suggest that Indonesian students continue to face substantial challenges in developing a sound understanding of fundamental mathematical concepts.

The relatively low level of mathematical proficiency among Indonesian students indicates that several mathematical topics continue to present significant learning challenges, one of which is algebra, particularly algebraic factorization. As a fundamental concept in algebra, algebraic factorization serves as a prerequisite for understanding more advanced mathematical topics (Oktavia & Yulia, 2025). Nevertheless, many students continue to experience difficulties in mastering this concept (Nurhana & Mulbar, 2025). Although the curriculum emphasizes the importance of contextual learning that is closely connected to students' everyday experiences in order to promote conceptual understanding and foster meaningful learning environments (Tasya & Syafi'i, 2025), classroom instruction often remains procedural and abstract. One promising approach to addressing this issue is the integration of local cultural contexts into mathematics instruction through the Ethno-Realistic Mathematics Education (Ethno-RME) approach. Ethno-RME integrates the principles of Realistic Mathematics Education (RME) with ethnomathematics, viewing mathematics not only as a human activity but also as a cultural practice (Prahmana et al., 2023). In the present study, the *Pinehala* customary sanction of the Tolaki Mekongga community in Kolaka Regency serves as the cultural context for learning. Within this customary practice, components of the customary compensation, such as buffaloes (*Kiniku* or *Karambau*), traditional cloth, and ritual expenses, can be represented algebraically using variables, coefficients, and constants. These representations provide students with meaningful opportunities to develop an understanding of algebraic factorization while simultaneously fostering an appreciation of their local cultural heritage.

Several previous studies have investigated the development of HLT based on cultural contexts as well as the integration of digital technologies into mathematics education. For instance, Kurnia et al (2023); Qatrunnada et al (2025); Sohilait & Sidik (2025) developed HLT using local cultural contexts as meaningful starting points for learning, including culinary traditions, the Masohi cultural practice, and *songket* weaving. In contrast, Ulya et al (2025); Wulandari et al (2025) focused on integrating digital technologies into mathematics instruction to enhance students' conceptual understanding, learning engagement, and twenty-first-century skills. However, their studies did not incorporate local cultural contexts into the design of HLT. Although these studies demonstrate the educational potential of either culturally contextualized learning or digital technologies, they remain limited to specific cultural contexts. To the best of the authors' knowledge, no previous study has developed a HLT that employs the *Pinehala* customary sanction of the Tolaki Mekongga community as the starting point for learning algebraic factorization in a digital classroom.

Accordingly, the present study makes two primary contributions to the existing literature. (1) it employs the *Pinehala* customary sanction of the Tolaki Mekongga community as an authentic cultural context to support students in developing a contextual understanding of algebraic factorization; (2) it proposes HLT specifically designed to accommodate the characteristics, technological resources, and instructional needs of digital classrooms.

METHODE

This study adopted a qualitative approach using the design research methodology. According to Gravemeijer & Cobb (2006), design research aims to simultaneously advance educational theory and instructional practice by designing, implementing, and systematically investigating learning interventions in authentic classroom settings. Through this process, design research seeks to generate a deeper understanding of students' learning processes and the instructional means that support them.

This study employed a design research methodology with the aim of designing and developing a mathematics learning trajectory based on the Ethno-Realistic Mathematics Education (Ethno-RME) approach through the development of a HLT. The research design followed the framework proposed by Gravemeijer & Cobb (2006), which comprises three main phases: preliminary design, teaching experiment, and retrospective analysis. However, the present study was limited to the preliminary design phase, focusing on the development and validation of the proposed HLT. This methodological scope is consistent with the approach adopted by Wangsa et al. (2025), who likewise conducted a limited design research study focusing exclusively on the development of an HLT.

Data were collected through classroom observations conducted during the preliminary design phase to examine the quality of mathematics instruction and identify challenges encountered in the implementation of digital classroom learning. These observations aimed to identify issues such as the underutilization of digital learning technologies and students' difficulties in understanding mathematical concepts. Furthermore, the proposed HLT was evaluated through Focus Group Discussions (FGDs) involving both mathematics education experts (university lecturers) and mathematics teachers as practitioners. The FGDs were conducted to obtain constructive feedback on the proposed learning trajectory and to evaluate its content validity, linguistic clarity, and contextual appropriateness, thereby ensuring that the HLT was suitable for implementation in mathematics classrooms.

RESULTS AND DISCUSSION

The findings from the preliminary study revealed that mathematics instruction in the digital classroom had not yet connected mathematical concepts to authentic contexts that were meaningful and relevant to students' everyday experiences. Instead, instruction remained predominantly abstract, with mathematical formulas introduced directly without first engaging students in a meaningful real-world situation as the starting point for conceptual development. This instructional practice highlights the need for a more meaningful and contextualized learning design, particularly to support students in developing a conceptual understanding of algebraic factorization. In response to these findings, a HLT was developed by integrating the *Pinehala* customary sanction as an authentic cultural context with students' progressive mathematical thinking through the iceberg model. The proposed learning trajectory was implemented through a Student Activity Worksheet (Lembar Aktivitas Siswa/LAS) designed to align with students' anticipated progression of conceptual understanding as specified in the HLT. In addition, student conjectures were formulated to anticipate the range of possible strategies, responses, and misconceptions that students might demonstrate while completing each learning activity. These conjectures served as a foundation for designing appropriate instructional support (scaffolding) throughout the learning process.

The selection of the *Pinehala* customary sanction as the starting point for learning algebraic factorization was based on several considerations. First, students' knowledge of and appreciation for the *Pinehala* customary sanction, which constitutes part of the local cultural heritage of the Kolaka community, have gradually declined. Therefore, incorporating this context into mathematics learning has the potential to reintroduce and preserve local cultural values. Second, the customary dispute resolution process within the *Pinehala* customary law possesses characteristics that make it an appropriate context for supporting students' gradual development of the concept of algebraic factorization. A review of the literature indicates that the *Pinehala* customary sanction includes several elements with the potential to serve as a meaningful context for mathematics learning. During the customary dispute resolution process, the required customary

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compensation consists of a buffalo, symbolizing the offender's responsibility; a white cloth (burial shroud), symbolizing respect for the victim; a water container, representing purification; and a machete, symbolizing honor (Febrian et al., 2024; Pairin et al., 2024; Saputra et al., 2020). These four components constitute an inseparable set of items that must be provided in accordance with the provisions of customary law. The amount of customary compensation required depends on the type of violation committed and therefore involves different combinations of these customary items in specified quantities.

Based on these characteristics, each compensation item can potentially be represented as an algebraic variable, coefficient, or constant. However, not all of these components were modeled as variables in the proposed mathematical representation. Instead, the researchers intentionally selected the buffalo and the machete to represent algebraic variables, while the white cloth and the water container were treated as fixed components that invariably accompany the customary settlement process. This selection was intended to simplify the mathematical model while preserving the cultural significance embedded in the implementation of the *Pinehala* customary sanction. The proposed HLT for teaching algebraic factorization, developed on the basis of this cultural context, is illustrated through the iceberg model presented in Figure 1.

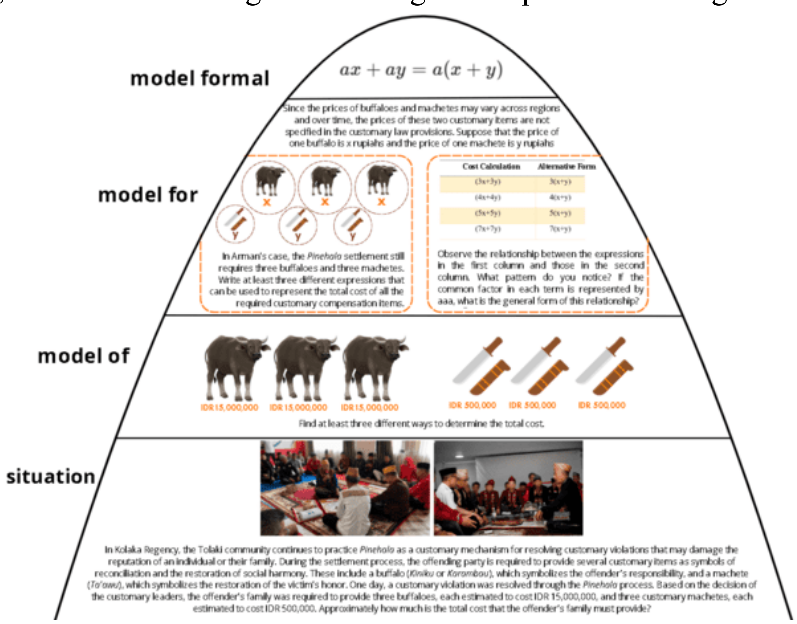

Figure 1. Iceberg Model of Algebraic Factorization Using the *Pinehala* Customary Sanction Context as the Starting Point

Based on Figure 1, the learning trajectory was designed as a sequence of interconnected stages that progressively linked the *Pinehala* customary sanction to the formal representation of algebraic factorization. Through the iceberg model, students were guided through a process of mathematization, beginning with horizontal mathematization, in which they explored and

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represented authentic situations using informal mathematical representations, and progressing to vertical mathematization, in which they generalized these representations to construct formal mathematical concepts (Surat et al., 2023). In this regard, Widianti & Setianingsih (2024) argue that mathematization encompasses two complementary forms of mathematical activity: informal mathematics, which is associated with horizontal mathematization, and formal mathematics, which is associated with vertical mathematization. This progressive process enables students to develop a deeper and more coherent understanding of mathematical concepts. At the situational level, students were introduced to the *Pinehala* customary sanction, in which the provision of buffaloes and machetes (*Ta'awu*) constitutes part of the customary compensation. Building on this context, students engaged in a sequence of carefully designed learning activities that guided them to represent the authentic situation using simple mathematical representations (model of), generalize numerical relationships into symbolic algebraic expressions using variables (model for), and ultimately construct the formal concept of algebraic factorization, expressed as $ax + ay = a(x+y)$ (formal model). Based on this iceberg model, a LAS was developed to operationalize the proposed learning trajectory, as illustrated in Figure 2.

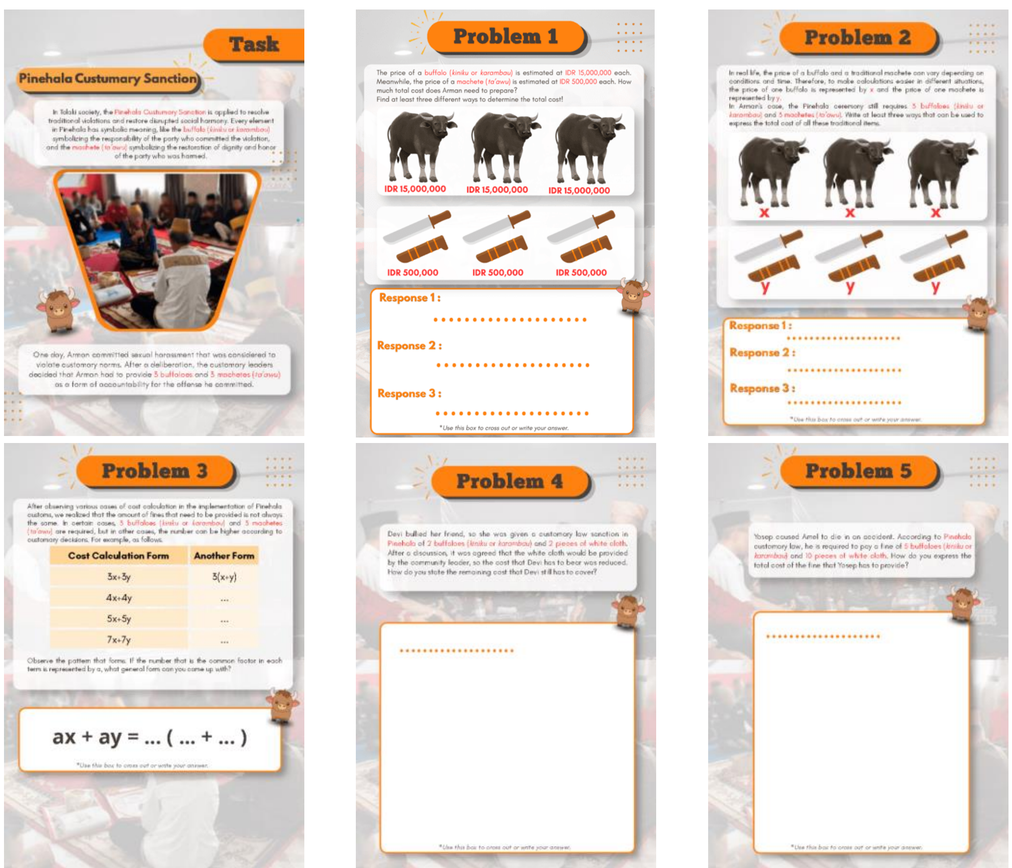

Figure 2. Student Activity Worksheet

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Figure 2 presents the Student Activity Worksheet (Lembar Aktivitas Siswa/LAS), which consists of six student activities, including one situational description and five mathematical tasks. The LAS was designed in accordance with the stages of the iceberg model, progressing systematically from the situational level to the formal model. To accommodate the characteristics of the digital classroom in which the study was conducted, the worksheet was developed in a digital format, as each student had access to an individual tablet for learning. The digital LAS was created using the Canva platform and implemented in the participating digital classroom. Canva was selected because it represents an innovative digital learning platform that supports the collaborative implementation of digital transformation in education (Riswanda & Rakhmawati, 2025). Through this platform, students were able to access and complete each activity sequentially in accordance with the proposed learning trajectory. Nevertheless, the integration of digital technology into mathematics instruction requires careful instructional design to ensure that technology enhances, rather than distracts from, students' learning. When students' attention is directed more toward the technological tools than toward the mathematical learning activities themselves, the process of mathematization may be disrupted (Julfan & Haifaturrahmah, 2025). Therefore, the digital LAS was designed not merely as a medium for delivering learning materials but as an integral component of the proposed HLT, supporting students' progressive construction of mathematical understanding. The anticipated student conjectures associated with each learning activity are presented in Table 1.

Table 1. Design of the Hypothetical Learning Trajectory (HLT)

Task: Presenting Information about the *Pinehala* Customary Sanction and a Contextual Case	
Anticipated Student Responses	-
Learning Activities and Hypothesis	Students become familiar with the ***Pinehala* customary sanction** as the instructional context and begin to identify mathematically relevant information embedded in the contextual situation.
Anticipated Student Difficulties	Students may focus primarily on the cultural narrative without recognizing the mathematical information embedded in the context.
Teacher's Instructional Support	The teacher poses guiding questions such as, *"What mathematical information can you identify from this situation?"* and encourages students to highlight the relevant information presented in the illustration.
Problem 1: Determining the Total Cost of Three Buffaloes and Three Machetes with Known Prices	

Anticipated Student Responses	**Response 1:** Calculating the cost of each item separately.. (3 × 15.000.000) + (3 × 500.000) = 45.000.000 + 1.500.000 = 46.500.000 **Response 2:** Calculating the total cost of one compensation package first.. (15.000.000 + 500.000) × 3 = 15.500.000 × 3 = 46.500.000 **Response 3:** Using repeated addition.. 15.000.000 + 15.000.000 + 15.000.000 + 500.000 + 500.000 + 500.000 = Rp. 46.500.000
Learning Activities and Hypothesis	Students determine the total cost of the customary compensation items by applying different yet mathematically equivalent solution strategies.
Anticipated Student Difficulties	**Response 1:** Students identify fewer than three different solution strategies. **Response 2:** Students incorrectly calculate $15{,}000{,}000 + 500{,}000$as $20{,}000{,}000$. **Response 3:** Students incorrectly apply the order of arithmetic operations.
Teacher's Instructional Support	The teacher encourages students to explore alternative calculation strategies while emphasizing that different procedures can produce the same mathematical result.
Problem 2: Determining the Total Cost of Three Buffaloes and Three Machetes with Unknown Prices	
Anticipated Student Responses	**Response 1:** $3x + 3y$ **Response 2:** $3(x + y)$ **Response 3:** $(x + y) + (x + y) + (x + y)$
Learning Activities and Hypothesis	Students represent the contextual situation using algebraic expressions.
Anticipated Student Difficulties	**Response 1:** Students write xyor $3xy$, assuming that $x$and $y$should be multiplied. **Response 2:** Students do not recognize that $x$and $y$represent the costs of two different compensation items.
Teacher's Instructional Support	The teacher reminds students that $x$and $y$represent the prices of different items. Students are then encouraged to relate each variable to its corresponding item in the context before comparing the resulting algebraic expressions.
Problem 3: Identifying Patterns and Generalizing	

Anticipated Student Responses	$3x + 3y = 3(x + y)$ $4x + 4y = 4(x + y)$ $5x + 5y = 5(x + y)$ $7x + 7y = 7(x + y)$ Students recognize the pattern $ax + ay = a(x + y)$ and conclude that the general form of algebraic factorization is $ax + ay = a(x + y)$
Learning Activities and Hypothesis	Students construct the general form of algebraic factorization through a process of pattern generalization.
Anticipated Student Difficulties	Students experience difficulty generalizing the factorization pattern from several examples.
Teacher's Instructional Support	The teacher asks students to compare several examples side by side and poses questions such as, *"Which part remains the same?"* Visual grouping models or diagrams are then used to highlight the common factor.
Problem 4: Applying the Concept	
Anticipated Student Responses	Initial cost: $2x + 2y$ Since the white cloth is no longer required, $2x + 2y - 2y = 2x$ Therefore, the remaining compensation cost that Devi must provide is $2x$
Learning Activities and Hypothesis	Students apply algebraic operations and factorization concepts to explain changes in a contextual situation.
Anticipated Student Difficulties	Students do not recognize that the entire cost associated with the white cloth has been removed.
Teacher's Instructional Support	The teacher asks students to relate each algebraic term to the corresponding compensation item and poses the question, *"Which part of the compensation has been removed?"* A visual list of the compensation items is then used to reinforce the meaning of subtracting like terms.
Problem 5: Applying the Concept in a More Complex Context	

Anticipated Student Responses	Total compensation cost: $5x + 10y$ Students identify **5** as the common factor. $$5x + 10y = 5x + (5 \times 2y) = 5(x + 2y)$$ Therefore, $$5x + 10y = 5(x + 2y)$$
Learning Activities and Hypothesis	Students apply algebraic factorization to solve a more complex contextual problem.
Anticipated Student Difficulties	(1)Students incorrectly identify the common factor; (2)Students factor only the first term; (3) Students fail to simplify $10y$to $2y$after factoring out the common factor.
Teacher's Instructional Support	The teacher encourages students to verify the common factor by dividing each coefficient by the proposed factor and asks, *"If 5 is factored out, what remains in each term?"* Students are then invited to substitute numerical values for the variables to verify that the original and factored expressions are equivalent.

Based on Table 1, the situational task introduces students to the *Pinehala* customary sanction, a customary dispute resolution practice in Kolaka Regency that requires the provision of buffaloes and machetes as components of customary compensation. At this stage, students are presented with a contextual case involving a sexual harassment incident committed by *Arman*. Using this authentic situation as the starting point for learning, students proceed to Problem 1 (model of), where they represent the contextual situation using simple mathematical operations by determining the total compensation cost. In this task, the price of a buffalo is assumed to be IDR 15,000,000 per animal, while the price of a customary machete is assumed to be IDR 500,000 per piece.

In Problems 2 and 3 (model for), students were guided to generalize the numerical relationships identified in the previous task by expressing them symbolically using the variables x and y, thereby constructing algebraic expressions that represent multiple possible situations. In Problem 3, students were asked to examine several algebraic expressions, such as 3x + 3y, 4x + 4y, 5x + 5y, and 7x + 7y, identify the common factor in each expression, and rewrite them in a simpler factored form, for example, 3(x + y). Through comparing these examples, students were expected to recognize the underlying pattern and, at the end of Problem 3 (formal model), generalize it into the formal algebraic factorization rule:ax + ay = a(x + y). Students were therefore expected to understand algebraic factorization as a generalized mathematical concept derived from the patterns they had previously identified, rather than as a formula introduced directly by the teacher. This progression reflects the transition from horizontal mathematization to vertical mathematization, in which students progressively move from contextual reasoning toward formal mathematical reasoning. Subsequently, Problems 4 and 5 presented different contextual situations to provide students with opportunities to apply the algebraic factorization concept they had constructed in Problem 3. These application tasks were designed to reinforce students' conceptual understanding by encouraging them to use the generalized factorization rule in new yet related contexts.

The proposed HLT was designed to support students in constructing mathematical understanding through authentic contexts rather than by introducing abstract concepts or formulas at the outset of instruction. This design is consistent with the findings of Marniati et al. (2026), who argued that mathematics instruction focusing primarily on formulas often hinders students' conceptual understanding because of its highly abstract nature. Similarly, Jahring et al. (2025) emphasized that connecting mathematical content to authentic real-world contexts facilitates students' conceptual understanding by making mathematical ideas more meaningful and accessible. Klorina & Prabawanto (2023) further reported that students' reliance on memorizing formulas frequently leads to errors in algebraic manipulation and contextual problem solving. Likewise, Rahayu et al. (2023) found that context-based mathematics instruction is more effective in promoting conceptual understanding than conventional formula-oriented teaching approaches. Taken together, these findings support the view that contextual learning, implemented through a gradual process of mathematization, plays an essential role in fostering meaningful mathematical understanding. By engaging students in authentic situations before introducing formal mathematical representations, the proposed HLT aligns with the principles of Realistic Mathematics Education (RME), in which students progressively reinvent mathematical concepts through guided learning experiences rather than receiving them as ready-made knowledge.

Addressing students' limited conceptual understanding of mathematics requires instructional approaches that emphasize meaningful learning experiences rather than procedural knowledge alone. One such approach is the integration of the iceberg model with the principles of Realistic Mathematics Education (RME). Within this framework, teachers are better equipped to anticipate students' thinking processes and prepare appropriate instructional support (scaffolding) throughout the learning trajectory. Consequently, learning is designed not only to guide students toward the intended mathematical concepts but also to accommodate the diverse strategies and misconceptions that may emerge during the process. Previous studies have likewise demonstrated that local cultural contexts can serve as meaningful starting points for the development of HLT. For example, Wangsa et al. (2025) employed a popular cultural context to support learning on arithmetic sequences and series, whereas Marniati et al. (2026) used the Tugu Api monument as the context for learning the arc length of a circle. Similarly, Qatrunnada et al. (2025) and Umasugi et al. (2022) both incorporated culinary contexts, although they addressed different mathematical topics, namely linear equations and social arithmetic. Meanwhile, Sohilait & Sidik (2025) dan Kurnia et al. (2023) developed HLT using different cultural contexts—Masohi culture and songket weaving, respectively—to facilitate learning on direct proportion. Collectively, these studies highlight the importance of instructional innovation that extends beyond improving learning outcomes to fostering students' mathematical thinking through structured and meaningful learning trajectories. In line with these findings, the present study contributes to the literature by introducing the *Pinehala* customary sanction as a novel cultural context within an Ethno-Realistic Mathematics Education (Ethno-RME) framework. The proposed HLT demonstrates how authentic local culture can be systematically integrated with digital learning environments to support students' progressive mathematization and conceptual understanding of algebraic factorization.

The use of the same context throughout the six learning activities provides continuity in students' learning experiences, enabling them to focus on the process of mathematization without having to repeatedly adapt to new contextual situations. This continuity of context helps reduce extraneous cognitive load, as students' cognitive resources are no longer used to understand different stories or situations but are instead directed toward identifying patterns, relationships among quantities, and developing mathematical concepts. Consequently, students have the opportunity to build conceptual understanding gradually through the exploration of the same context with progressively increasing levels of activity complexity. This is consistent with Cognitive Load Theory, which explains that learning becomes more effective when irrelevant cognitive load is minimized, allowing working memory capacity to be focused on knowledge construction (Klepsch & Seufert, 2020).

The consistent use of the same context also supports the process of guided reinvention, a fundamental principle of Realistic Mathematics Education (RME) that positions students as active rediscoverers of mathematical concepts through systematically designed learning activities (Van den Heuvel-Panhuizen & Drijvers, 2020). Each activity is not designed as an isolated task but rather as a continuation of the preceding one, enabling students to progressively develop informal strategies toward more formal mathematical representations. This process reflects a coherent learning trajectory, in which each activity serves as a bridge to the subsequent conceptual understanding. According to Van den Heuvel-Panhuizen & Drijvers (2020), meaningful contexts function as the starting point for the mathematization process while simultaneously supporting the development of a systematic learning trajectory from informal strategies to formal mathematical concepts.

The integration of Canva further strengthens the implementation of the proposed learning trajectory by presenting learning activities in a more visual, interactive, and structured manner. Features such as images, step-by-step illustrations, simple animations, and digital response fields enable students to manipulate information more easily than with conventional instructional media. The visual representations provided by Canva assist students in identifying patterns, connecting contextual representations with mathematical representations, and facilitating the transition from concrete understanding to abstract reasoning. From the perspective of Realistic Mathematics Education (RME), the use of multiple visual representations serves as a model that bridges students' informal thinking and formal mathematical (Van den Heuvel-Panhuizen & Drijvers, 2020). Furthermore, the use of interactive digital media can enhance students' engagement and support more meaningful exploration of mathematical concepts (Trouche & Drijvers, 2014).

During the learning process outlined in the HLT, scaffolding plays an important role in supporting students when they encounter difficulties at each stage of the learning activities. Support is provided progressively through guiding questions, reminders of information obtained from previous activities, simplification of problems, and prompts that encourage students to independently identify relationships among quantities. Scaffolding is not intended to provide students with direct answers but rather to support the development of their thinking skills until they are able to solve problems independently (Chen et al., 2023; DiNapoli & Miller, 2022; Vogel et al., 2022). As students' understanding improves, the level of support is gradually reduced (fading), allowing them to construct mathematical concepts based on their own reasoning. This strategy is consistent with the concept of implicit scaffolding, which emphasizes the provision of prompts, feedback, and activity designs that encourage student exploration without diminishing their opportunities to construct knowledge independently (Podolefsky et al., 2014).

If viewed from the perspective of the characteristics of Realistic Mathematics Education (RME), the proposed learning design demonstrates alignment with several of its key principles. First, the use of a single context that is closely related to students' experiences reflects the principle of phenomenological exploration, in which meaningful situations serve as the starting point for learning. Second, the sequentially designed activities facilitate progressive mathematization, enabling students to move from informal strategies to symbolic representations and the generalization of mathematical concepts (Van den Heuvel-Panhuizen & Drijvers, 2020). Third, the use of tables, visual illustrations, and the Canva platform reflects the principle of models and representations, as these models function as a bridge between real-world contexts and formal mathematical concepts. Fourth, classroom discussions, student presentations, and teacher scaffolding exemplify the principle of interactivity, whereby learning occurs through interactions among students as well as between students and the teacher. Finally, each activity connects previously acquired concepts with new mathematical ideas, thereby demonstrating intertwinement, which refers to the interconnectedness of mathematical concepts within a coherent learning trajectory (Van den Heuvel-Panhuizen & Drijvers, 2020)

Overall, the combination of the consistent use of a single context, the support provided by the Canva platform, the implementation of adaptive scaffolding, and the design of learning activities aligned with the principles of Realistic Mathematics Education (RME) creates a learning environment that supports students in developing conceptual understanding progressively. This design not only facilitates the achievement of the intended learning objectives but also strengthens students' mathematization process through meaningful, structured, and contextual learning experiences.

CONCLUSSION

This study successfully developed a HLT for teaching the concept of algebraic factorization in a digital classroom by using the *Pinehala* customary sanction of the Tolaki Mekongga community as the starting point for learning. The HLT was designed based on the Ethno-Realistic Mathematics Education (Ethno-RME) approach through the iceberg model, which guides students to develop conceptual understanding progressively, beginning with a contextual situation, followed by mathematical representation (model of), generalization (model for), and culminating in the formal concept of algebraic factorization. The learning design was implemented through a digital LAS consisting of six learning activities and was complemented by student conjectures that predicted students' strategies, responses, and possible thinking processes at each stage of the learning trajectory. The *Pinehala* customary sanction context provided meaningful learning experiences by enabling students to connect local cultural phenomena with mathematical concepts through the processes of horizontal and vertical mathematization. Consequently, students not only learned the procedures of algebraic factorization but also constructed the concept by identifying patterns emerging from authentic situations. The integration of the local cultural context, the iceberg model, and digital media within the HLT offers an alternative instructional design that supports mathematics learning which is more contextual, systematic, and aligned with the characteristics of digital classrooms. This study contributes to the development of mathematics learning design based on Ethno-RME, particularly through the use of the *Pinehala* customary sanction of the Tolaki Mekongga community as a context for learning algebraic factorization. The resulting HLT can serve as a reference for teachers in designing culturally responsive mathematics instruction in digital classrooms and as a foundation for future research to examine the implementation and effectiveness of the proposed design in improving students' conceptual understanding across various school contexts.

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