Students' Development In Notating The Unit Of Speed

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ABSTRACT:

Students have acquainted with speed from their environment but it is not easy for them to learn the concept of speed since according to Gravemeijer (2007) , the concept of speed is still one of the most difficult quantities. What students learn about speed will be developed when they study at higher education it is very important for teacher to give support for the students so that they are not only able to use the formula deal with the speed but also know the reason behind the formula and how to apply it in other situations. Hence, a series of learning activities which in line with Realistic Mathematics Education (RME) was designed. Design research was chosen as the general heading of the research. It was conducted in SD Al-Fatah(Full day School) in Surabaya-Indonesia with a teacher and 25 students in fifth grade. Research findings show that "paper tapes activity" can lead the students to develop their notion on the unit of speed. Students realise that in order to compare the speed they must be aware of both distance and time as the components of the speed. The function of paper tapes which the students made gradually changing in line with students' thinking process. At the first time, it might only as the path for the toy cars, but then it shows the distance travelled by the car. Finally, students modified those paper tapes as the visualization of the unit of speed which help them to made a table as a strating point to come to the other units of speed.

Key words : *RME, Design research, Unit of speed, Paper Tapes Activity*

INTRODUCTION

Students have acquainted with speed from their environment. Most of the time they move either by walking or running, they use bicycle, motorcycle or car to go to the school, they involve in a kind of race competition and so forth. In school, students learn the concept of speed in the upper grade of primary school. However, it is not easy for them since according to Gravemeijer (2007) the concept of speed is still one of the most difficult quantities. It may because speed is a compound quantity then children must pay attention simultaneously to two quantities as they learn this concepts, distance and time. However, children have the tendency to look primarily speed at the distance travelled and lose sight of the time that is required to travel this distance.

Nevertheless, what students learn about speed will be developed when they study at higher education not only for mathematics subject but also in science, especially in physics. Regard with this, it is very important for teacher to give support for the students so that they are not only able to use the formula deal with the speed but also know the reason behind the formula and how to apply it in other situations.

This research focuses on supporting fifth graders in learning the concept of speed as well its unit. In order to reach it, the researcher designs a series of learning activities which in line with Realistic Mathematics Education (RME). The researcher names one of the activities as "paper tapes activity" inspired by the paper used by the students as the path of the toy cars. The researcher uses toy cars **because we can keep the speed in constant and students are familiar with it. It is also possible for students to investigate the speed of toy cars directly. Using her design,** the researcher tries to create a learning environment in which she can observe how students notate the unit of speed, so does their thinking and reasoning behind. Hence this research is underlined by a research question : *How can paper tapes activity bring students to develop the unit of speed?*

THEORETICAL FRAMEWORK

The Concept of Speed

In everyday usage, the terms 'speed' and 'velocity' are interchangeable (Serway, 2009). We may also wonder the use of letter instead of for speed. Cassidy et.al, (2002) explain that the concept of speed is closely related to the concepts of velocity, from which the symbol arises. Speed is perceived as the property of an object that is relative to the ground (Doorman, 2005). Speed indicates how fast something is moving regardless of whether or how it may change the direction. In present study, we use term speed instead of velocity since we only concern with speed as the quantity of motion and this is what the students learn in elementary schools.

Unit of measurement is a standardized quantity of a physical property. In order to make accurate and reliable measurements, people use units of measurement that do not change and can be adopted by observers all over the world. These units have been internationally approved and called standard units. Since speed is a derived quantity which is derived from the base quantities, distance and time, then based on the International System of units or SI the units of speed is a combination among the units of distance(length) and time namely kilometre per second (km/h), meter per second (m/sec) and so forth.

Realistic Mathematics Education

Realistic Mathematics education is rooted in Freudenthal's interpretation of mathematics as an activity (Freudenthal, 1991). Mathematical activity involving solving problems, looking for problems and organizing a subject matter and *mathematizing* as the main activity (Gravemeijer., 1994). It was Treffers who placed the two ways of *mathematizing*. He distinguished 'horizontal' and 'vertical' *mathematizing*. In the case of horizontal *mathematizing*, mathematical tools are brought forward and used to organize and solve a problem situated in daily life. Vertical *mathematizing*, on the contrary, stands for all kinds of reorganizations and operations done by students within the mathematical system itself (Van den Heuvel-Panhuizen, 2003). The central principle of RME is that mathematics should always be meaningful to students. The term 'realistic' stresses that problem situation should be 'experientially real' for students. This does not necessarily mean that the problem situations are always encountered in daily life. Students can experience an abstract mathematical problem as real when the mathematics of that problem is meaningful to them (Bakker, 2004).

RME offers heuristic or principles for instructional design in mathematics education ;

guided reinvention, didactical phenomenology and emergent modelling (Gravemeijer, 1994). The following session will describe these heuristics.

Guided reinvention

The reinvention principle in RME states that students should be given the opportunity to experience a process similar to the process by which mathematics was invented (Gravemeijer, 1994). It means that it allows the learners to come to regard the knowledge they acquire as their own private knowledge, knowledge for which they themselves are responsible. In this present study, we expect that students reinvent the unit of speed that involving the unit of distance and time as the way to quantify something moves. The reinvention principle can be inspired by the history of mathematics and the informal procedure. In order to support the reinvention principle, the appropriate contextual problem that allow for a wide variety of solution procedures need to considered.

Didactical Phenomenology

A didactical phenomenology refers to looking for the situation that creates to be organized by the students. Such an analysis investigates how the concepts we want to teach help organise this situation, and how they can be problematical for the students (Doorman, 2005). The phenomena "beg to be organised" is used as the starting point for education. We use the phenomenon of measuring speed of humans or animals moves in daily life as the starting point since one cannot use a tool like a speedometer to measure but indeed, he can use the combination of apparatus for measuring distance and time to measure the speed.

Emergent Modelling

In the third principle, self-developed model play as the bridge of the gap between the informal knowledge and formal mathematics (Gravemeijer, 1994). In RME, it is expected that the students develop the models by themselves. Materials, visual sketches, paradigmatic situations, schemes, diagrams and even symbols can serve as model (Treffers and Goffree, 1985; Treffers 1987, 1991; Gravemeijer, 1994a in Van Heuvel - Panhuizen, 2003). The models first appear as a *model of* the situation that means very close to the problem situation at hand. Then gradually it generalized over situations become a model for mathematical reasoning to foster higher-level ideas.

The concept of speed is a type of imaginary situation as we explained before. Here students need materials, visual sketches, schemes, and so on to organize the situation. In present research, we will never know exactly what kinds of models that will appear. However, the sophisticated models for the concepts of speed are models for proportional reasoning, which are the ratio table and the double number line because they both offer a systematic way of writing down the relation between distance and time (Gravemeijer, et al 2005).

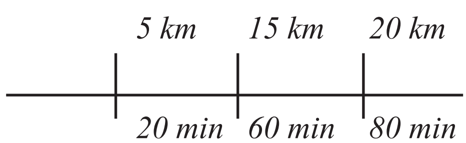
A ratio table is a mental model as well as a work sheet. It plays an important role in the curriculum on proportion (Gravemeijer, 2007). Ratio table is an ideal aid for making handy calculations and gaining insight because the table invites students to write down intermediate steps. Moreover, they explain that the strength of the ratio table is that students can reason with number relationship that they already know. To calculate how long someone will take to cycle 20 km at an average speed of 15 km/h, one can do as:

Km	15	5	20
Minutes	60	20	80

From the table above we can see clearly every intermediate steps and what they mean. In this case, 15 km in 60 minutes means the same as 5 km in 20 minutes. From the table also students might immediately see that we can go from 15 to 20 with 5 as an intermediate step. Other students might need more intermediate steps to calculate from 15 km to 20 km so that he/she makes a ratio table as shown below:

Km	15	30	10	20
Minutes	60	120	40	80

Another model that can be used to explain the concept of speed and it relations between distance and time is double number line. A double number line has the same principle with a ratio table but the position of points in double number line is meaningful. This model allows students to link distance and time together; it is allow students to make accurate calculations and estimates as well. To calculate how long someone will take to cycle as the problem above, a double number line will show as follows:



Above we can see that the intermediate steps that had been used is actually same with the first example in the ratio table; 15 km in 60 minutes means the same as 5 km in 20 minutes hence multiply the proportion of 5 km in 20 minutes by 4 would get 20 km in 80 minutes. All of the models show the same result in which with an average speed of 15 km/h someone need 80 minutes to travel 80 km of distance.

Ratio table and the number line model can work together for the concept of speed and it relations between distance and time. However, these models are not familiar in Indonesia even in mathematics textbook so that we conjecture that it is difficult for students to reinvent by themselves. Nonetheless, the contexts used in the learning process give students opportunity to make the model of the situation. Afterward, teacher can provoke them to make a table that has the same principle with a ratio table but more in vertical forms since students' book provide the lines in horizontal way so that make a vertical table is easier and more natural than a horizontal table.

RESEARCH METHOD

In order to reach the research aim and answer the research question, a sequence activity with Indonesian context was designed to improve educational practises in learning the concept of speed for five graders of elementary school. Hence, the present research falls under the general heading of design research. According to Bakker (2004) a design research consists three phases as following:

1. Preparation and design phase
2. Teaching experiment

3. Retrospective analysis

In the preparation phase, the relevant theory about the topic was studied. The theory came from various sources in mathematics, science, realistic mathematics education, design research and teaching experiment as the initial conjectures for the path of the learning process. Then, Hypothetical Learning Trajectory (HLT) was designed. It consists of the goal for the students' learning, the mathematical tasks that will be used to promote students learning, and hypotheses about the process of the students' learning.

This research consists of two cycles. In every cycle, the researcher apply those phases. At the teaching experiment of first cycle, the researcher tried out the initial HLT in a small group of the students then compared with the actual learning process so there are some revision and reformulation of the initial HLT which is applied on the second teaching experiment.

The second teaching experiment conducted in SD Al-Fatah (Full Day School) Surabaya on second semester of the academic year 2010/2011 during three weeks (eight lessons) with 70 minutes of the duration for each lesson. A mathematics teacher and 25 students in fifth grade involved in this research. The purpose of this phase is both to test and to improve the conjecture of the learning trajectory that was developed in the previous phase and to collect the data for answering the research questions.

In order to investigate the learning process and to collect the data, the researcher uses video recording and written data. The researcher uses two cameras; one camera as a static camera to record the whole class activities and the other camera records the discussion in the group of students. There are also videos of the interviews with the teacher and the students. The written data consists of students' work in solving the problems during the class activity and in the final assessment including observation sheets and some notes gathered during the teaching experiments.

HYPOTHETICAL LEARNING TRAJECTORY

Simon in Simon and Tzur (2004) offered the Hypothetical Learning Trajectory (HLT) as a way to explicate an important aspect of pedagogical thinking involved in teaching mathematics. As the researcher mentioned before that an HLT consists of the goal for the students' learning, the mathematical tasks and hypotheses about the process of the students' learning. After tried out the initial HLT, the researcher modified some activities and some tasks for improving the HLT. The overview of revised HLT as following :

Activity : "Paper Tapes Activity 1"

Learning Goals :

- Students can compare the speed of the car by measuring the distance in a unit of time
- Students can analyse distance and time growth

Mathematical task :

The problems are given orally to the students, they are asked to

- Measure the time and the distance travelled by the car

- Find the way to compare the speed of the toy cars
- Determine the distance travelled by the toy cars for every second

Conjectures of students' thinking:

1. Students will not get difficulties in measuring the time and distance travelled by the car.
2. - Students propose to use the same distance
   - Students propose to use the same time

Activity : "Paper Tapes Activity 2"

Learning Goals :

Students compare the speed of six toy cars by notating the speed in a standard unit

Mathematical task :

- Notate the speed of the toy car
- Make a table of the distance and time of the car travelled in several times

Conjectures of Students' thinking :

1. Students notate the speed in standard or non standard units
2. Students use the table in vertical or horizontal forms

RESEARCH FINDINGS AND ANALYSIS

As the starting point of the lesson, teacher divided the students into six groups. There are four or five students in a group. Every group had a toy car, a measuring tape, a stopwatch and a single track. The lengths of every track given to the students are different. Students then asked to measure the distance and the time of their car. For the time, teacher asked to omit per hundred seconds. Students did the investigation outside the class as shown in the following figure

Figure 1. Students measured the time and the distance of the toy cars

All of the students came together to the tracks that have been prepared before. They worked cooperatively. None of the group got difficulties in measuring the distance and the time. After doing the investigation, students wrote the results on their paper tapes. Below is the result of their investigation:

Table 1. Students' Investigation results

NO	GROUP	DISTANCE	TIME
1	Group 1	150 cm	3 seconds
2	Group 2	250 cm	5 seconds
3	Group 3	350 cm	6 seconds
4	Group 4	450 cm	6 seconds
5	Group 5	500 cm	9 seconds
6	Group 6	600 cm	13 seconds

In order to provoke students come to measure the distance in a unit of time, teacher asked the students to find the way to compare the speed of all of the toy cars so that they can determine the fastest car. No student proposes his/her answer. Its imply that although students have the data it is still very difficult for them to find the fastest car. The teacher then explained that he would use the same time for all and the time that he choose is one second. Hence, the lesson continued with predicting the distance of each toy car in one second.

It is also not easy for the students to predict the distance in one second. After some discussion in a several time, group 6 (Kelompok 6) wrote and counted the distance divided by the time for toy cars in group 1 and 2 but they didn't write the unit of the calculation result. This group also decided that toy car in fourth group is the fastest and the toy car in the first and the second have the same although again he can't give the reason behind the calculation. Same with group 6, the students in group 4 (Kel 4) devided the distance by the time to get the number then they wrote "in every 1 second" behind the result of their calculation. This group only finished calculating four toy cars. Students in group 1, 3 and 5 finished to calculate the distance travelled by the car in one second. Meanwhile students in group 2 (kelompok 2) only rewrite the distance and time from the investigation without changed it into per second. We can see the students' works as folloeing

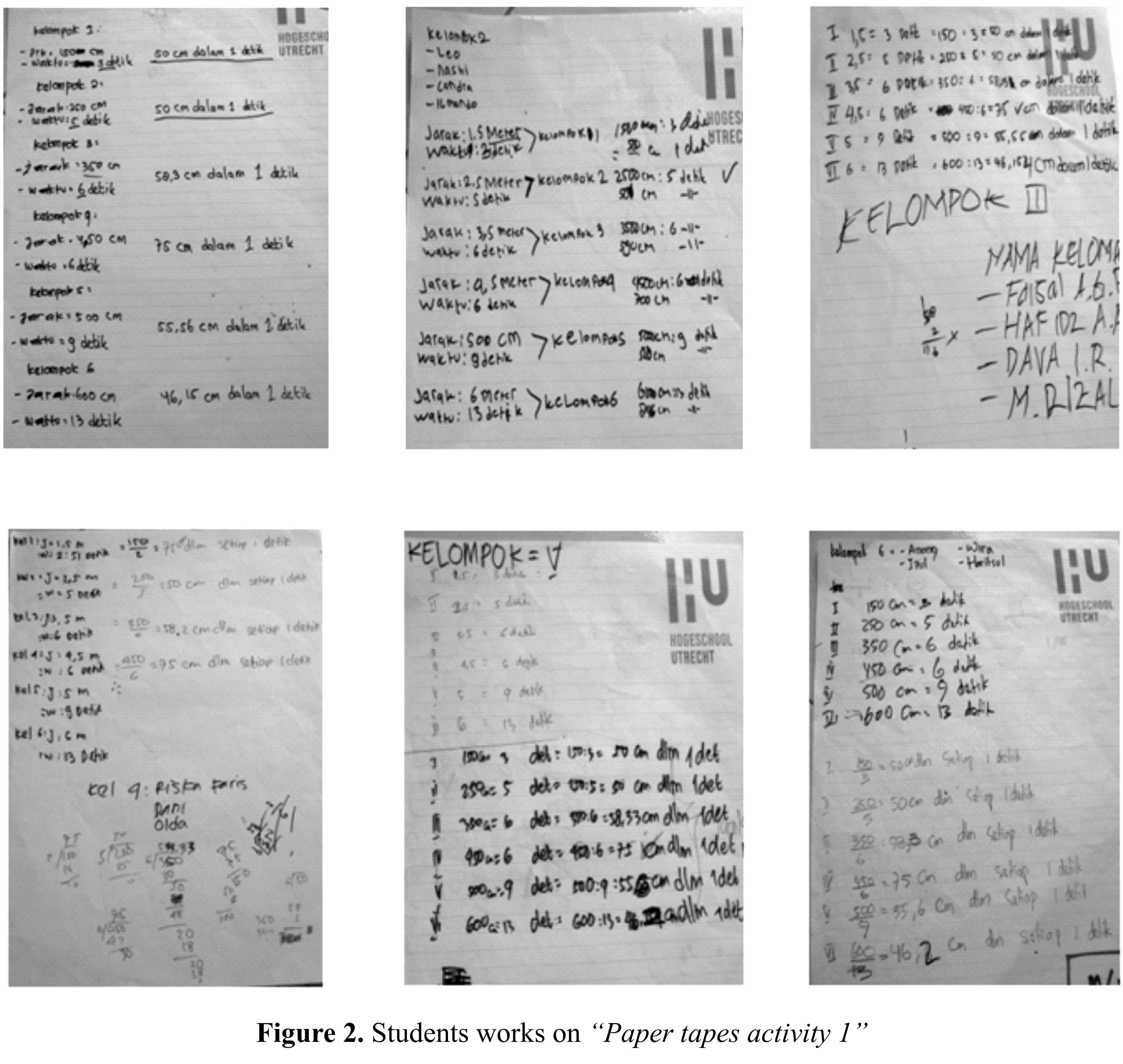


Figure 2. Students works on *"Paper tapes activity 1"*

From the students work, we can see that although teacher didn't give the formula of speed, students divided the total distance by the time. Students' calculation actually shows the speed of the toy cars. Unfortunately, no students realize that it was “speed”. Students in this activity didn't write explicitly the unit of speed in a standard unit yet. Nevertheless, most of the students explained that 50 above means 50 cm for every second. This explanation will be the base for the meaning of certain distance in a unit of time which is reference to the unit of speed.

In this activity, students not only calculate the distance travelled by the car in a second, but they also put their car in the track as if the cars travel for a second. First, students measured the distance that they want using a measuring tape. The distance that they used come from their calculation before. After that, they put the car on it. Without request from the teacher, group 4 made a sign in the paper tape for the distance travelled by the car in one second.

Figure 3. Group 4 measured the distance of the car during one second

Knowing what group 4 did, teacher asked other groups to make signs of distances travelled by their car for every second. All of the groups measured the distance without doing real investigation. For the first sign (students make lines as the sign), they measured the distance based on their calculation. After that, they only continued by measuring the same distance using prevous line as the starting point.

Figure 4. Group 1 made signs for the position of the car for every second

At the next activity, there was a class discussion deal with the paper tapes that students made. Teacher sticked all of the paper tapes on the wall so that every students know other groups' work. The discussion focused on the growth of the distance and time as if the toy cars travel until the end part of the paper tapes. Knowing the distance in one second, students immediately show the distance in two seconds, three seconds and so on from signs (lines) in paper tapes. These lines divided the paper tapes into some parts, which is the same as the total time of the movement. Since students made the lines by themselves, they realized that the length of every segment is equal. The following vignette is a part of the discussion:

Teacher	:	*Start from group 1, what is the distance for every one second?*
Students	:	*50 cm*
Teacher	:	*For 2 seconds?*
Students	:	*100* (centimetres)

Teacher : For 3 seconds?
Students : 150 cm

(a few minutes later)
Teacher : What about group 2, what is the distance of it (for every one second)*?*
Students : 50
Teacher : Group 3
Students : 58,3
Teacher : So it is faster (than the first and the second group)
What about this, group 4. Riska. What is the distance for every one second?
Students : 75 cm
Teacher : So, it is faster right. What about this group?
Students : 55,6
Teacher : What about this [point to the paper tape of group 6]?
Students : 46,2
Teacher : So, which one is faster?
Students : Group 4
Teacher : Why?
Students : Because it travels for 75 seconds in every second

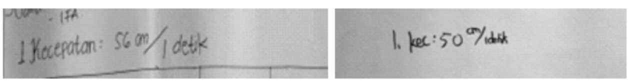


Figure 5. Students notate the speed in a standard unit

After finishing the discussion, teacher asked the students to write the speed of their toy cars. It was not an obligatory for the students to write in a standard unit. Nevertheless, all of the students use an International system to notate the speed, namely in cm/sec, below are some of students' work in notating the speed.

The notion of cm/second might be inspired by the length unit of the paper tapes and the unit of time that they used before (students use cm as the unit of the length and second for the time). It is also easy for students to show the distance for every second because they did the investigation before. However, students need a discussion to come to the notion of *"cm/second"*. The discussion mostly about how to interpret daily life language into mathematics language.

Next, every group made a table of distance and time based on their investigation. This table shows implicitly the proportion among the distance and time. When teacher asked the students to extend the table as long as they want, students didn't get any difficulties.Moreover, students come to other units of speed namely cm/minute, m/minute, km/h and cm/h.

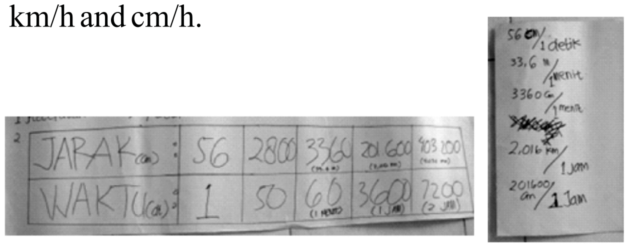


Figure 6. Group 1 notated the speed in various units

CONCLUSION AND DISCUSSION

In summary, we can say that "Paper Tapes Activity" can lead the students to develop their notion on the unit of speed. In this activity, students realise that in order to compare the speed they must be aware of both distance and time as the components of the physical quantity of speed. Moreover, comparing speed of many objects provoke them to use the same unit of speed and one way is measuring the distance travelled by the car for every one unit of time as the base of the unit of speed. The function of paper tapes which the students made gradually changing in line with students' thinking process. At the first time, it might only as the path for the toy cars, but then it shows the distance travelled by the car. Finally, students modified those paper tapes as the visualization of the unit of speed. Using the paper tapes, students can see how the distance and time growth during the movement of the toy cars. Knowing the this growth, students didn't get any difficulties to write their investigation in a table as well to extend it so that they come to the other units of speed. Throughout the research, we found that at the first time many students get difficulties to show the unit of speeed. However, teacher could help them by doing discussion either in a group or in a class. Using a discussion, there are exchange ideas among teacher and students so that students are accustomed to share ideas to others.

This article may give an overview for teachers in elementary school how to conduct a lesson about the speed in a mathematics classroom. “Paper Tapes Activity” is only one of the examples. Teachers can modify and develop it in other situations. They might use other activities as long the lesson is “students centered”. Students know the speed from their environtment and what we did in the class is develop this knowledge. The more they involve actively in the lesson, the more meaning that they get.

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