THE THINKING TRAJECTORY PROFILE OF STUDENTS TO PROVE THEOREM IN REAL ANALYSIS SUBJECT

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Abstract

This study is a descriptive study aimed to describe students' thinking trajectory in proving theorem within real analysis I course. The subjects of this study are two students, man and woman who both have high academic ability based on GPA for courses in mathematics. The research instrument consists of researcher, interview sheet, interview guides and student worksheet that contains proving theorem. In this research the data analysis technique are data pruning, data presenting and conclusing. From the data analysis, it is obtained that the there is different thinking trajectory from the male and female subjects. The male student's thinking trajectory: 1) At the information input stage, the subject understands well what is known and what will be proved, 2) at the information processing stage, the subject is able to construct the proving steps correctly, the subject uses the definition available before, 3) within the information output step, the subject does not recheck the steps that he has written down, the subject gives answer illustration through pictures but he faces difficulty in explaining the picture. Female subject's thinking trajectory : 1) at the information input stage, the subject understands well and she is able to write what is known and what is will be proved correctly, 2) at the stage of information processing, the subjects writes long and coherent steps , but there is a concept that has not been understood well , in constructing the proof, she does not use the existing definition, 3) at the information output stage, the subject re-checks the evidentiary steps which she writes previously, subject give answers with pictures and illustrations which she can explain well.

Keywords: thinking trajectory, proving theorem

INTRODUCTION

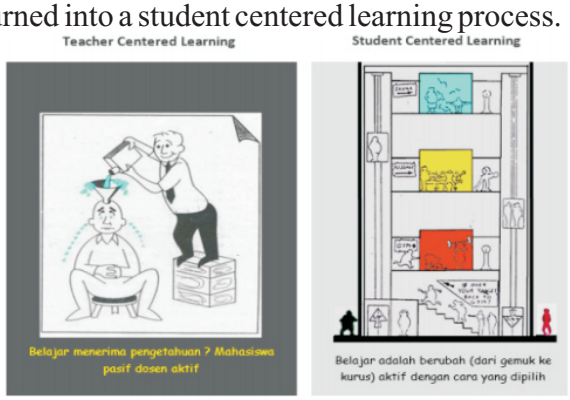
One of the important parts in teaching is knowing the students' way of thinking (Shulman, 1987). From preparing lessons, asking questions to facilitating discussion in the classroom can figure out the students' way of thinking (Henningsen& Stein, 1999). Students does not only use knowledge but they are also required to create or develop knowledge. In learning mathematics students must learn with an active understanding, build their own new knowledge based on experience and existing knowledge (NCTM, 2000). It can be prepared by every individual through the best education. Within the interest of achieving optimal education, improving the quality of education must be done sustainably by all relevant parties including the government.

The Government has a lot of efforts to improve the quality of higher education. One of the ways was to issue a regulation of the ministry of education and culture of the Republic of

Indonesia Number 73 2013 about the implementation of Indonesia National Qualifications Framework (KKNI) of higher education. Once, a regulation of ministry of education and culture No. 49 2014 was also issued about National Standards for Higher Education, the department is required to produce graduates with KKNI qualification. Based on NCTM (2000) curriculum is just not a collection of activities; it must be systematic, concentrated on important mathematics, and well articulated across the grades.

In KKNI -based curriculum, the learning process is encouraged to focus on student (Student centered learning / SCL) by focusing on expected learning outcomes . Centered on the student means that graduates' learning outcomes is achieved through a learning process that promotes the development of creativity , capacity , personality , and the students' needs and develops the independence in searching and finding knowledge .

This is in line with the theory of Piaget. Piaget's theory has a lot of reactions on the instructional design. The Learning process which was previously centered on the faculty turned into a student centered learning process.



Picture 1.1 TCL and SCL

One of the factors that support the development of cognitive psychology is the development theory of Piaget. According to Piaget's theory, the structure of human thought to adapt to the environment is through two mechanisms: assimilation and accommodation. Definition of Assimilation is integrating process from new information directly into a pattern that has been formed. While the accommodation is the process of integrating new information by changing the old scheme or the formation of a new scheme to fit with the information that will be accepted.

According to Piaget, learning is emphasizing the acquisition of information and the development of a person's knowledge. One's learning process will follow its pattern and stages of development according to age. These pattern and stages are hierarchical, meaning that must be passed by a certain sequence and someone cannot learn something that is beyond the cognitive stage.

Knowledge cannot be moved away from lecturers to students, but the students themselves construct knowledge. In constructing knowledge, students should be able to dig up information or knowledge they have learned previously and past learning experiences to find knowledge.

Mathematics is a basic science to develop science and mathematics technology. It's main product is in forms of statements such as definitions, theorems, result, conjecture ,dll. The accompanying numbers and arithmetic operations is the mathematical derivative products (Hernadi , 2013) .Statements in

mathematics such as theorem and result need to be validated.

Wadsworth (1989 in Suparno, 2001 : 141), remembering and memorizing are not considered as real learning because these activities do not include the process of assimilation and comprehension . A child who knows the name of numbers does not mean that he understands the concept of the numbers. Remembering is included in mental activity in proving a theorem. That is why, the trajectory in proving a theorem which is meant by the researcher is the thinking trajectory.

The term trajectory of learning was first used by Simon (1995) for designing learning. The trajectory of the study provides an overview of the activity sequence in achieving learning goals. The same term (learning trajectory) used by Chuang (2002) to examine how mental activity sequence on solving the problem is. According to Piaget (Sumarno, 2001)learning emphasizes more on the acquisition or the increase of information or knowledge. According to Mayer (Solso, 1988) thinking is an activity that is directed to produce a solution.

The thinking trajectory in proving theorem gives an image of the part where the difficulty faced by the students happens. This knowledge will help us in determining the way how to parse the difficulty. The trajectory in proving theorem will illustrate the level of someone's thinking level of understanding of the primary material, as well as the concept or other knowledge required to prove the theorem.

HIP (Human Information Processing) analogizes the human thought process as a computer work process consisting of three stages. The first stage is to get ininformation, the second stage is information processing and final stage is the deliverance of the information that has been processed or output. (Morgan, et al. 1986).

The process of thinking is a mental process that exists in the human brain to process certain information. Hence, the thinking process will involve cognition. The process of cognition refers to the whole process in which the sensory input is changed, interpreted, stored, recalled and used. According to Neisser (in Ben R. Newell, 2008) state that a brainy organism operates in a perception action cycle: the senses take in information from the surrounding, the mind/brain that performs computations on information and the outputs of reviews those computations are used to guide subsequent goal-directed actions, which means that the cycle that occurs on organism in processing information is an organism receives information from the outside, then the brain processes / performs calculation to the information and the result of the calculation will be the basis for taking the next step. So it is clear that cognition is involved in the whole things that may be done by humans.

Proof is a series of arguments or logical steps that explain the truth of a statement. One of the statements that must be proved in mathematics is theorem. Theorem can be in a form of sentence that includes a relationship with one or more premises and a conclusion (Hernadi , 2013).

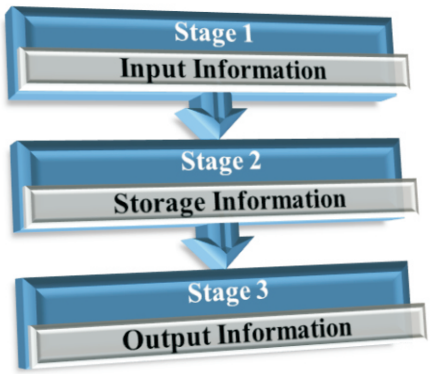
Students are still experiencing difficulties in proving theorems, especially in the

subject of real analysis. Therefore, students need to be involved directly in proving the theorem so that students can understand and apply the theorem correctly.

Proving the theorem is demonstrating systematic and deductive - logical steps to convince someone about the validity of a statement/theorem (Siswono and Rosyidi, 2010). By proving theorems, students can learn a logical expression to express the validity of a statement or a theorem.

According to Chuang (2002) the thinking trajectory is individual. Every student has different characteristic, including the ability to solve problems of mathematics as well as proving theorems, this is due to several factors including gender difference. The gender difference results in differences in the structure of the left brain and right brain between boys and girls. This difference can also result in traits differences such as attitude, motivation, talent and performance. In this study the researcher uses the thinking trajectory which is made up of three stages, namely the acceptance of information (input), the information processing (storage) and the processed-information deliverance (output).

Krutetski in Anisa (2011) stated that men were superior in logical reasoning while women were superior in accuracy , precision , carefulness , and thoroughness in thinking. Men have the ability in mathematics and mechanics better than women. Proving theorem requires the ability to think logically so it needs reasoning. Hence, the level of ability and gender difference are the factors considered in this study as well, especially in choosing the subjects.



Picture 1.2 The stages of thinking trajectory

METHOD

This research is descriptive qualitative research. The subjects are selected from students who take the course of Real Analysis I. The selection of the subject considers the level of mathematical skills of the students measured by GPA of math courses. In this study, the selection of the subjects is limited only to students with the high academic ability. For this reason two students with the best GPA for each gender are selected.

The research instruments consist of the researcher herself , student worksheets , interview guides and interview sheet. Data analysis technique used in this study aredata pruning, data presenting and conclusing. In line with Miles and Huberman (1994) stated that a qualitative analysis of data were described by the data reduction, data presentation, and verification.

RESULT AND DISCUSSION

Data collection was performed twice, on 17 and 18 December 2015. The research subjects were two students who took the course of real

analysis with high academic ability. The subjects consist of one male student and a female student. Here is the data of the research subject:

Table 3.1 Reseach Subject

No	Initial name	Sex	Symbol
1	OLM	Female	S_1
2	BFT	Male	S_2

Once the subjects were chosen, on 10 December 2015, the researcher gave the worksheet formerly to the subject prior to the interview. The worksheet is as follows:

Arrange the proof for the following theorems

2.4.1 Let S be subset of R

i. An element $u \in R$ is said to be an upper bound of S if $s \leq u$, for all $s \in S$

ii. An element $w \in R$ is said to be a lower bound of S if $w \leq s$, for all $s \in S$

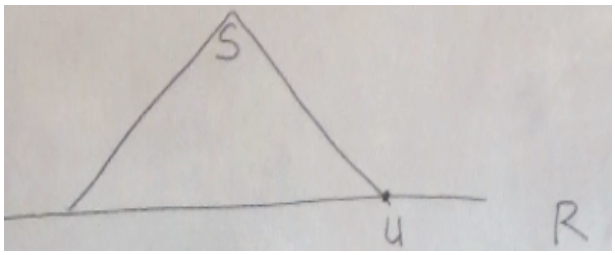
2.4.2 Let S be subset of R

i. If S is bounded above, then an upper bound is said to be a supremum (or a least upper bound) of S if it is less than every other upper bound of S.

ii. If S is bounded below, then a lower bound is said to be an infimum (or a greatest lower bound) of S if it is greater than every other lower bound of S.

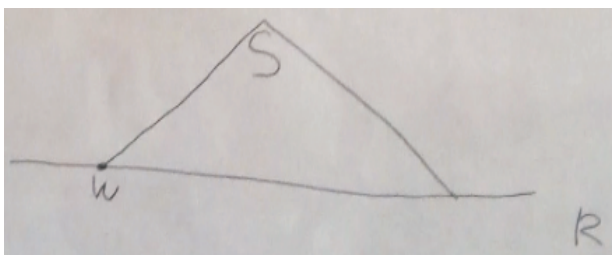
Furthermore, on 17 December 2015 the researcher conducted interview to subject to prove the theorem 2.4.1. The researcher conducted in-depth interview to the subject to obtain the students' thinking trajectory in proving the theorem. Here is the transcript of the interview toward the research subject.

S1 : " Let S be the subset of R, then the first one there is an u element , It is called as the upper bound if u is greater than or equal to S for every s element S. Suppose there is a real number line



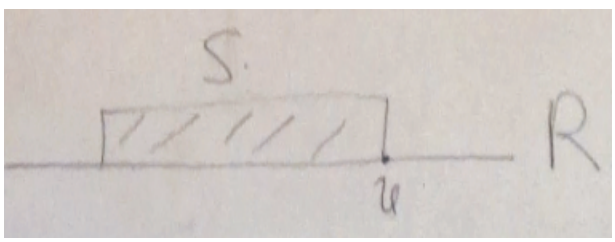
Picture 3.1 Illustration S_1 for upper bound of S

Means u is the upper bound of R (pointing to the picture) secondly, suppose w is element R, w is the lower bound if w is less than or equal to s for each s element S.



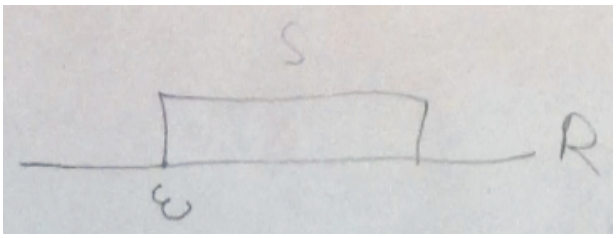
Picture 3.2 Illustration S_1 for lower bound of S

S2 : " An element of a ... , let S be subset of R , then it is said to be upper bound if there is x element R until x is less than or equal to s , where s element S. for more details the explanation:



Picture 3.3 Illustration S_2 for upper bound of S

This x is here (pointing to the picture), x is upper bound because x is greater than all the elements of S. Further for the lower bound, if there is w element R until w is less than or equal to s, where s element S.



Picture 3.4 Illustration S_2 for lower bound of S

W is located here (pointing to the picture), w is smaller than every element of S"

From the results of these interviews, it could be seen that the thinking process of the subjects, the S1 and S2 subjects could write down the proving steps fluently and correctly was because the subjects had quite a long time to learn and to construct the steps of the proof, the worksheet had been given on the previous week as a project.

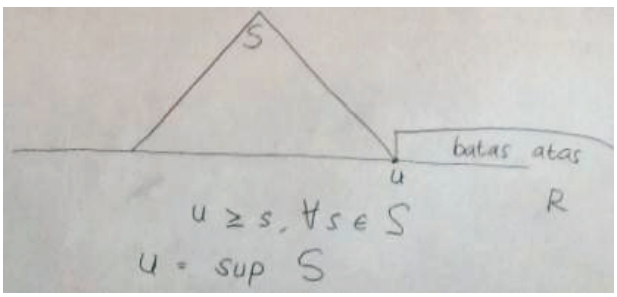
Thinking trajectory of the students are presented in the following table :

Table 3.2 Thinking trajectory of the subjects

Subject code	First trajectory	Second trajectory	Third trajectory
	Information input	Information processing	Information output
S_1	Subject understood what is known and what would be proved	Proved a theorem using a proving planning, but there were still a few mistakes in writing down the answers. Subjects wrote steps very coherently as the procedure Subject could provide the proof with illustration through pictures.	Before drawing conclusions, subject retraced the steps of work that have been done. The steps made in the proving step were correct
S_2	The Subject understood what was known and what would be proved	The subject did the proving process more briefly. Subject headed directly to what was being asked. Subjects could give illustration through images.	The subject did not recheck the proving steps which has been done. All steps done were correct.

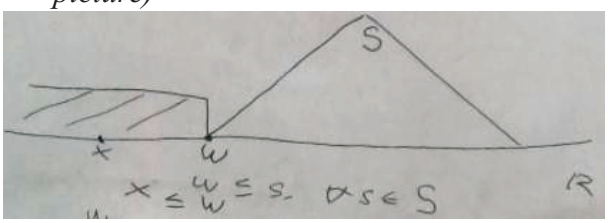
On December 18, 2015 the researcher conducted an interview to subjects to prove the theorem 2.4.2 Here is a transcript of the interview to the subject.

S1 : " Suppose there is S , S is a subset of R , S is bounded above if a ... (silent) ... the upper bound is called as supremum / least upper bound if S is smaller than the bound (then the subject drew the proof)



Picture 3.4 Illustration S_1 for supremum of S

u to there is the upper bound (pointing to the picture) , u here is the lowest upper bound, so u is a supremum . Suppose v is taken as another example of upper bound, then v is greater than u , the smallest one here is u , then u is the supremum of S. Furthermore, if s is the lower bound, then lower bound is the infimum / or least lower bound of the smallest of S .. .a ... (long pause) ... if the lower bound is bigger than other boundaries . (The subject then made a picture)

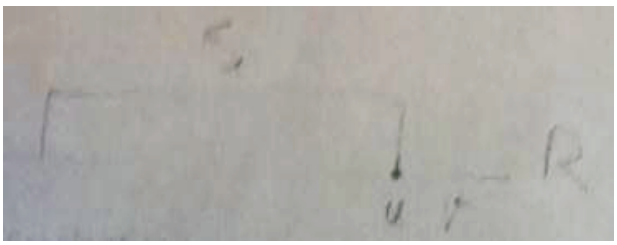


Picture 3.5 Illustration S_1 for infimum of S

(Pointing at the picture) suppose other boundaries are taken , eg x then x is less than or equal to w , so w is the greatest one"

S2 : "ok I will proving this theorem, first….we know that S is bounded, then the least upper bound of S is called as supremum . To find the supremum, it is back to the initial definition, so we determine whether there is an upper bound first then we seek the supremum itself. We take any v as the upper bound of S , then the definition used is x , x is less than or equal to v,

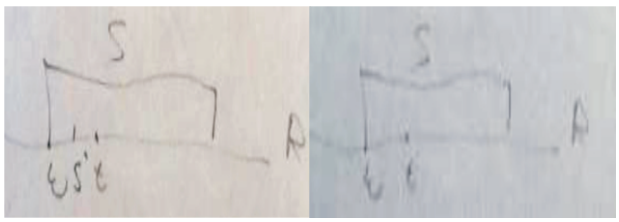
$S = \{ v \in R \mid v > s, \forall s \in S \}$ *where* $x < v$ *(while drawing)*



Picture 3.6 Illustration S_2 for supremum of S

x is less than, then it is called as supremum. Further, like the supremum, take any t as the least bound of S. So, t is less than or equal to v,

$$S = \{t \in R \mid t \leq s, \forall s \in S\}, \text{where } x \leq v, \text{where } t \leq w$$



Picture 3.7 Illustration S_2 for infimum of S

(Researchers asked why t was there while pointed at the picture , it was paradoxical that t was smaller than all the s element S) . S2: "Yes because there is s which is smaller than t , then t is not the infimum of S , so the infimum of S is w "

For the theorem 2.4.2, from the result of the interview, the subjects' process of thinking could be seen. Subject S1 wrote the proving steps which were coherent, appropriate with procedure, but the subject did not use the definition 2.4.1, She provided good illustrations in the form of images . For subject S2 started the proof by using the definition 2.4.1, steps were written coherently and correctly, but when the subject gave illustrations for infimum, illustration given was less appropriate . The thinking trajectory of students is presented in the following table:

Table 3.3 Thinking trajectory of research subjects

Subject code	First trajectory	Second trajectory	Third trajectory
	Information input	Information processing	Information output
S_1	At first, the subject faced difficulty in understanding what would be proved. There was a long pause to understand the theorem.	The Subject directly constructed the proving steps without using the definition 2.4.1 The steps were arranged coherently and correctly. The Subject could illustrated the proof through pictures.	Before drawing a conclusion, subject retraced the work steps that had been done. Subjects could infer it correctly. The subject did not
S_2	The subject understood what was known and what would be proved. Subject could make a plan of the proving ways that was using the definition 2.4.1	The subject proved the theorem in a more brief way. The Subject headed directly to what is asked. The subject could give illustrations trough images. The first illustration for supremum was correct, the second one for infimum, the subject faced difficulty, there was a long pause when he illustrated the second one. There was an incorrect concept, the picture given did not match with the explanation.	recheck the proving steps that had been done. The conclusion taken was correct even though there was a mistake within the provided picture.

From the results above, it could be seen that the subject S1 and S2 faced obstacles in the process of proving theorem .For the subject S1, when preparing evidence for the definition 2.4.1, S1 wrote the proving steps very smoothly, but there was a concept that was less understood by S1, they were the concept of upper bound and bounded above concepts.

Subject S1 said that u was bounded above, while what would be proved was u as the upper bound of the S set which was bounded above, although S1 gave a proper illustration. Meanwhile, the subject S2 started the explanation with a correct sentence, what he would prove was S set which was bounded above, so the subject S2 could distinguish the concepts of upper bound and bounded above, but for the theorem 2.4.2, S2 did errors when he gave an illustration in the form of images, S2 understood what was to be proved , but S2 faced difficulty in explaining through the images .

The Failure faced by the subject to resolve the problem was influenced by many factors , one of them was less accuracy of the subject and the prerequisite concepts were less mastered .According to Purnomp (2014) The factors that cause errors of students in solving the problem was 1) Students were not thorough in understanding the problem, 2) An error in selecting and using the concept of problem solving , 3) Errors on calculation. Based on Hudoyo (2003) a plan, understand the idea of the solution was not a guarantee to easily succeed , prerequisite knowledge and good mental habits were needed. Meanwhile, according to Polya (1973) another thing that might be a cause not to be success in solving a problem was to work with forgetting the plans that have been made.

One of the successes of the subject in proving theorems was through making systematic work steps, noting the important points needed , learning deeper the prerequisites concept . In accordance with the results of Daniel Long and David Carlson (2011) who stated that thinking maps are beneficial to students and teachers because they illustrate how students connection ideas and perceptions. Besides Torregrosa and Quesada (2008) stated that perceptual apprehension is characterised as the simple identification of a configuration. This is the first stage in the student's cognitive process.

CONCLUSION

Based on the results and discussion explained above, it can be concluded that:

a) High ability female subject (S1)
   At the information input stage of the subject understood the information well , he was able to write what was known and what would be proved correctly ; at the stage of information processing steps, the subject wrote long and coherent steps,but there was a concept that has not been well understood , in compiling evidence, the subject did not use the existing definition, At the output stage; the subject rechecked the proving steps which she had written, the subject illustrated the answers through pictures and she could explain it very well.
b) High ability male subject (S2)
   At the information input stage, the subject could mention information that he got in sequence, what would be proved. At the stage of information processing subject could arrange the proving step correctly, subject used the definition that existed before, at the information output stage, the subject did not recheck the steps he had written, the subject illustrated the answer with pictures but he faced difficulty in explaining the images.

SUGGESTION

Based on the result of this study, the researcher gives some suggestions as follow:

a) The subject with high ability does not always succeed in solving problems, there are many influencing factors, thus for further researchs, it is needed to involve subjects from various level; from low, middle to high is needed.
b) Further research about analysis of students' error in proving theorem and its influencing factors is required.
c) In designing a learning process needs to pay attention to the thinking trajectory of students to obtain optimal result.

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