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Profile of Students' Conceptual Understanding of Geometric Transformation Material Reviewed from Students' Learning Readiness

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ABSTRACT

This study aims to describe students' conceptual understanding of geometric transformation material in the aspects of explanation and application, viewed from their learning readiness. This research used a descriptive qualitative method involving three students with medium learning readiness selected from 35 students in class XI.8 of SMAN 1 Gresik. Subjects with developing learning readiness were chosen because most students in the class were at this level. Data collection was conducted through cognitive diagnostic assessments to determine students' learning readiness, particularly their mastery of prerequisite materials. Formative tests in the form of Student Worksheets and application tests were used to obtain data on conceptual understanding, supported by semi-structured interviews. Data analysis followed qualitative research procedures, including data reduction, data presentation, and conclusion drawing. The results showed that students with developing learning readiness met all indicators in the explanation aspect (accurate, justified, and systematic). However, in the application aspect only some students met the effective indicator, and only one student met the fluent indicator. The student with the best performance demonstrated structured and logical mastery of geometric transformation procedures, while the others still experienced difficulties in matrix calculations and applying concepts fluently. The findings suggest that teachers should provide more application-based problems after students construct concepts through guided discovery activities.

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INTRODUCTION

Mathematics serves as a basic science that serves as the foundation for various disciplines, making it a very essential subject in the education system (Makramalla et al., 2025; Nugraha et al., 2024). According to the Ministry of National Education in Regulation No. 22 of 2006, the main objective of learning mathematics is for students to understand mathematical concepts, explain the relationships between concepts, and apply concepts or algorithms flexibly, accurately, efficiently, and appropriately in problem solving. Mathematics is a tool for thinking, communicating, and solving problems (Santos-Trigo, 2024). The ability to reason, logic, creative thinking, problem-solving skills, and other mathematical abilities can be developed through mathematics (Gusteti & Neviyarni, 2022).

Understanding the concepts of a subject is fundamental to problem-solving (Mayasari & Habeahan, 2021). Conceptual understanding in mathematics is a crucial foundation that influences students' ability to develop logical thinking skills, solve problems, and apply mathematical knowledge in everyday life (Braithwaite & Sprague, 2021). In the learning process, the emphasis is on mastering students' conceptual understanding (Jäder & Johansson, 2025), ensuring students understand the material taught and have a solid foundation for achieving other basic skills such as communication, connection, reasoning, and problem-solving (Yanti et al., 2019).

However, difficulty understanding mathematical concepts remains one of the biggest challenges in education, both at the elementary and secondary levels. Many students simply memorize formulas or solution steps without truly understanding the underlying concepts (S. R. Hidayat & Warmi, 2024). As a result, they experience difficulties when faced with problems that differ from previously studied examples, which require a deeper understanding. A strong conceptual understanding not only supports mastery of the material but also helps students develop critical and analytical thinking skills that they can apply in various situations (Asterhan & Resnick, 2020; Barta et al., 2022; Dessie et al., 2024). Based on observations in class XI. 8 of SMAN 1 Gresik on the topic of geometric transformations with matrices, it was found that many students experienced difficulty in writing and performing matrix multiplication operations correctly. These errors indicate a problem in understanding the concepts of matrices and geometric transformations, which are often considered complex and abstract by most students.

One alternative that can be implemented to minimize these problems is differentiated learning (Jannah et al., 2022). Differentiated learning is an important way of thinking about the teaching and learning process in the 21st century (Qomari et al., 2022). According to (Qomari et al., 2022), differentiated learning allows educators to view learning from multiple perspectives, thus enabling educators to devote full attention and provide actions to meet the specific needs of students. Previous research conducted by Ervina (Eviana, 2023) showed that the application of differentiated learning in mathematics learning can improve conceptual understanding.

There are three important aspects as students' learning needs in differentiated learning according to Tomlinson (Tomlinson, 2017) as quoted by Salsabila (Salsabila et al., 2024), including: 1) learning readiness, namely students are ready with new material to face the next learning process; 2) learning interest, namely students have their own motivation in driving the desire to learn; and 3) learning profile, related to language, health, cultural, environmental and family conditions, and other specific factors. The first aspect of students' learning needs is students' learning readiness. Learning readiness is an important requirement for continuing the next learning process. In the context of differentiated learning, learning readiness is related to the initial abilities possessed by students (Fauziyah & Rahma, 2025). Students' cognitive abilities also have an influence on students' conceptual understanding. Conceptual understanding emphasizes students' cognitive abilities. Each student has a different level of ability (Aseptianova et al., 2019). Therefore, understanding students' learning

readiness levels is crucial for teachers to design learning that meets their individual needs (Fauziyah & Wahyunita, 2025). The purpose of identifying or mapping students' learning needs based on their level of readiness is to modify the difficulty level of the learning materials to ensure that students' learning needs are met.

The purpose of identifying or mapping students' learning needs based on their level of learning readiness is to modify the level of difficulty of the learning materials, thus ensuring that students' learning needs are met. In this study, Wiggins & McTighe's conceptual understanding theory was used as a basis for measuring the extent to which students understand concepts. According to Wiggins & McTighe, conceptual understanding consists of six main aspects: explanation, interpretation, application, perspective, empathy, and self-knowledge (Wiggins & McTighe, 2005). Each of these aspects represents a different way to demonstrate a student's understanding of a concept. If a student is able to demonstrate one of these six aspects, then the student is considered to have demonstrated a level of understanding. In this study, the researcher will focus the measurement of conceptual understanding on two of these six aspects: explanation and application.

In Understanding by Design (UbD), the explanation aspect refers to students' ability to provide clear, in-depth, and coherent explanations of concepts or ideas learned, so that they not only remember facts but also understand their meaning more comprehensively. Students are said to be able to explain when they are able to describe an idea in their own words, build relationships between topics, demonstrate work results, explain reasons/methods/procedures, explain a theory using data, argue and defend their opinions. Students can explain by 1) through generalizations or principles, 2) providing justified and systematic phenomena, facts, and data, and 3) making deep connections and providing illuminating examples or illustrations (Rahadi et al., 2023). Meanwhile, students are said to be able to apply a concept when they are able to effectively apply, use and adapt what is known in diverse and real contexts (Rahadi et al., 2023). True understanding is not just knowing the theory, but also the ability to adapt and apply concepts or skills to face new problems or situations.

Previous research has shown that gender differences significantly influence students' conceptual understanding in mathematics, where studies found variation in conceptual mastery by gender, including in mathematical problem solving and understanding of core concepts like the Pythagorean theorem (Azizah et al., 2021; Lestari & Palupi, 2023). For female students, the indicator of presenting concepts in various forms of mathematical representation was only partially fulfilled, but the other indicators were fulfilled. Inhibiting factors included arithmetic operation errors, conclusion errors, and lack of accuracy. Meanwhile, research (Komariyah et al., 2018) on the analysis of conceptual understanding in solving mathematical problems viewed from student learning interests showed that students with high learning interests were able to solve problems coherently and correctly. Students with moderate learning interests tended to rely on strategies without linking them to concepts, while students with low learning interests only restated concepts without being able to continue solving problems due to difficulties in planning and implementing strategies. Meanwhile, in this study, students' conceptual understanding abilities were reviewed based on their learning readiness. This is a novelty compared to previous research.

Based on the above, this study was conducted with the aim of describing the conceptual understanding ability of geometric transformation material in the aspects of explanation and application in students with moderate cognitive abilities. Learning readiness consists of advanced learning readiness, moderate learning readiness, and learning readiness requiring guidance. However, this study focused on one learning readiness, namely moderate learning readiness. The selection of subjects in the moderate learning readiness category because subjects with this readiness dominate in

mathematics classes. The benefit of this study is to increase knowledge and contribute ideas regarding students' conceptual understanding ability based on learning readiness.

METHODS

This type of research is descriptive qualitative. This research describes the profile of conceptual understanding ability in geometric transformation material in the explanation and application aspects in terms of learning readiness, but is limited to moderate learning readiness. According to Sugiyono (Sugiyono, 2012) qualitative method is an in-depth and comprehensive research approach to understand and explain phenomena in their natural context. The researcher acts as the main instrument, while the supporting instruments are (1) cognitive diagnostic assessment, which is used to determine students' initial abilities so that students can be grouped based on their learning readiness, (2) formative test sheets in the form of LKPD in the form of descriptions to determine conceptual understanding ability in the explanation aspect (3) application test questions, which are used to determine conceptual understanding ability in the application aspect. This research was conducted at SMA Negeri 1 Gresik, Gresik Regency, East Java, in November 2024 with 35 participants of grade XI.8 students. The students were given a cognitive diagnostic assessment related to prerequisite material to determine learning readiness groups. This assessment refers to the Independent Curriculum diagnostic assessment, with results categorized into three groups: proficient, moderate, and needing guidance.

After the learning readiness group was determined, three students were selected from the middle category group using purposive sampling to become research subjects. Purposive sampling, also known as judgment sampling, is a deliberate selection of participants based on their qualities (Etikan et al., 2016). Furthermore, the conceptual understanding ability in the explanation aspect of the three students was analyzed based on the results of a formative test in the form of a Student Worksheet (LKPD). Meanwhile, the application aspect was analyzed by correcting the results of the application problem solving test based on the alternative answers that had been created. The indicators used to measure students' conceptual understanding ability in the explanation and application aspects according to (Wiggins & McTighe, 2005) can be seen in the following table.

Table 1. Conceptual Understanding Indicators (Wiggins & McTighe)

No	Aspect	Indicator	Explanation
1	*Explanation*	*Accurate*	Able to explain their knowledge in their own words, based on facts, and accurately.
		Justified	Able to express their knowledge based on their own opinions, in detail, and can support their knowledge based on the literature.
		Systematic	Able to explain a concept systematically using structured and logical sentences.
2	*Application*	*Effective*	Students are able to achieve desired results using the knowledge or skills they have learned.
		Fluent	Students demonstrate good mastery, resulting in smooth implementation and error-free implementation.

The data analysis technique used was qualitative data analysis. The qualitative data analysis technique used was the Miles & Huberman (M. B. Miles et al., 2016) model of data reduction, data presentation, and conclusion drawing. Data analysis in this study was broadly divided into two stages: analysis of answer sheet data and analysis of interview results. Each of these data was analyzed alternately based on the research subject. The subject's answer sheet was analyzed step by step

according to the stages carried out. In this study, the analysis stages were based on stages according to indicators of conceptual understanding.

Miles and Huberman (M. Miles et al., 2018) state that "Qualitative data analysis was described by data reduction, data display, and conclusion drawing or verification." This means that qualitative data analysis is carried out through data reduction, data presentation, and conclusion drawing or verification.

Miles and Huberman (1994: 10) state that "Qualitative data analysis was described by data reduction, data display, and conclusion drawing or verification." This means that qualitative data analysis is conducted through data reduction, data presentation, and conclusion drawing or verification. Therefore, to analyze task-based interview data, the following stages were carried out: data categorization, data reduction, data presentation, and conclusion drawing. Data categorization in this study was based on indicators of conceptual understanding, which were then further categorized based on the type of understanding revealed.

In the data reduction stage, data selection, simplification, and transformation were carried out in the field. Data reduction in this study refers to the process of selecting data related to indicators of conceptual understanding. Furthermore, the selected and identified data were coded to identify the source of the data. The next stage is data presentation, namely writing down a collection of organized and categorized data so that it is easier to interpret the data to draw conclusions from the data.

FINDINGS AND DISCUSSION

This study aims to determine students' conceptual understanding abilities based on their learning readiness. However, in this study, the learning readiness used was limited to moderate learning readiness. Participants were given a cognitive diagnostic assessment to determine their learning readiness category. The results of the learning readiness diagnostic assessment are shown in Table 2 below.

Table 2. Cognitive diagnostic assessment results data

No	Readiness Level	Students
1.	Advanced Learning Readiness	5
2.	Moderate Learning Readiness	25
3.	Learning Readiness Needs Guidance	5

After identifying their learning readiness groups, three students were selected from the middle category using purposive sampling to become research subjects. The formative test and application test results from these three students were then analyzed, followed by semi-structured interviews. The analysis was divided into two sections: explanation and application. The following is a summary of the analysis.

Explanation aspect

In the accuracy indicator, subject 1 (S1) was able to explain his knowledge in his own words, according to the facts, precisely. Subject S1 wrote down the definition of geometric transformation composition correctly using his own language, although there were some inappropriate sentences, such as "many events." The results of subject S1's completion can be seen in Figure 1 below.

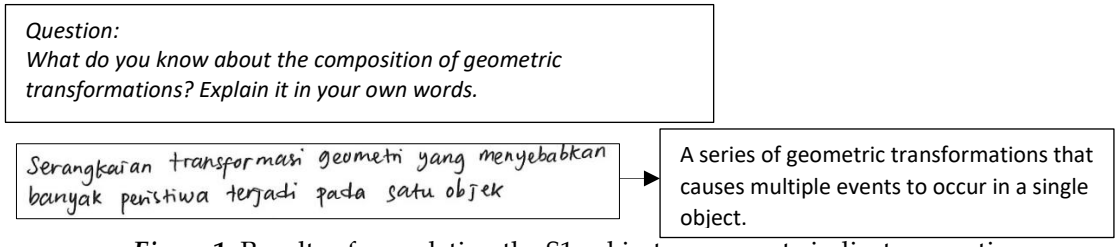


***Figure* 1**. Results of completing the S1 subject on accurate indicator questions

In the justified indicator, subject 1 (S1) was able to express his knowledge based on his own opinion, in detail, and could prove his knowledge according to the literature. Subject S1 wrote down the changes that occurred accurately, but there were slight errors in the form of the transformation. The interview results showed that to ensure the answers that had been written were correct or not, subject S1 used estimates and based on pre-existing knowledge and adjusted them to the literature. Figure 2 displays the results of the completion of subject S1 and Table 3 displays the interview transcript on the justified indicator.

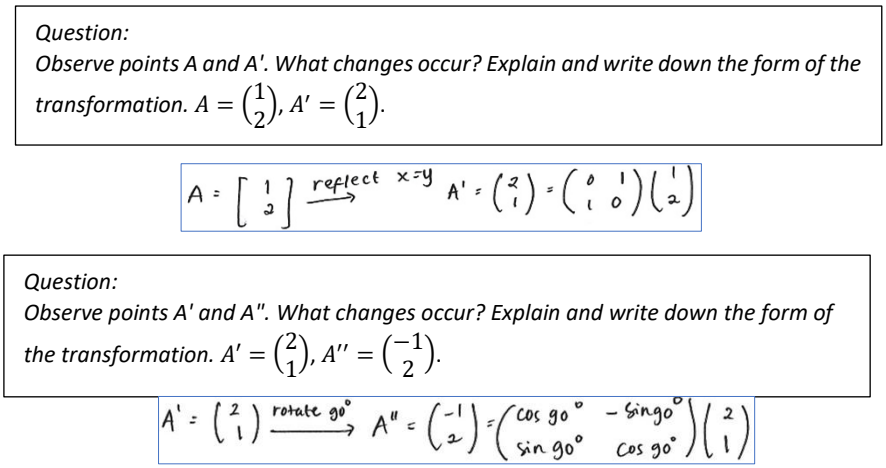


Figure 2. Results of completing the S1 subject on justified and systematic indicator questions

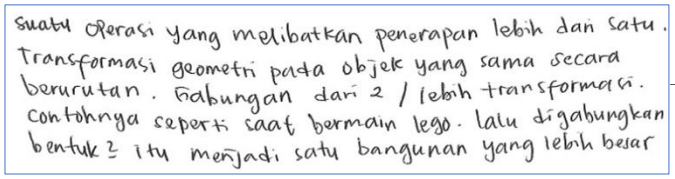
In the systematic indicator, subject 1 (S1) was able to explain concepts systematically using structured and logical sentences. Based on the results of the work and interviews, S1 was able to explain the concept of geometric transformation well. The subject demonstrated the ability to explain the steps of transformation, such as reflection on the line y = x and rotation by 90° with the center at (0,0), in a structured manner. In addition, S1 used logical arguments and supported his explanations with predicted results and references to relevant geometric transformation concepts. The following is a transcript of the interview with S1.

R: "Which are the starting point, the image point, and the transformation matrix?"
S1: "The starting point is $A = \binom{1}{2}$*, and the second point after reflection is* $A' = \binom{2}{1}$*, where (2,1) is obtained from the matrix* $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ *multiplied by* $\binom{1}{2}$*"*
R: "Is that written correctly?
S1: "I think it's written the other way around. After reflection, $x = y$ *is written as the matrix first, and the starting point is then equal to the result."*
R: "How do you know it's the other way around?"
S1: "If it's written like the one above, the answer will suddenly appear. If we don't know A', we automatically work on the right side first."

R: "How do you ensure that the change from A to A' is a reflection about y=x, and the change from A' to A'' is a 90° rotation with the center (0,0)?"
S1: "So, even if we don't use GeoGebra, we can predict the answer using the known starting and ending points, based on what we have learned, that reflection $x = y$ is just the image being reversed, and the most likely answer is reflection. We can also try it on a Cartesian diagram, drawing the starting and ending points, from there we can look for references for which geometric transformation is most suitable, whether it is reflection, rotation or others."

In the accuracy indicator, subject 2 (S2) was able to explain their knowledge in their own words, based on the facts, and precisely. Subject S2 wrote down the exact meaning of geometric transformation composition using their own words and could exemplify it using a frequently encountered game. The results of subject S2's completion can be seen in Figure 3 below.

Question:
What do you know about the composition of geometric transformations? Explain it in your own words.

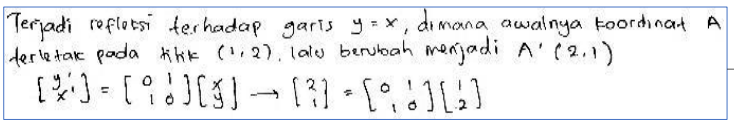


An operation that involves applying more than one geometric transformation to the same object in sequence. A combination of 2 or more transformations, for example when playing with Lego, then combining the shapes into a larger building.

Figure 3. Results of completing the S2 subject on accurate indicator questions

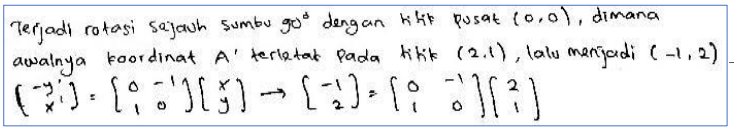
In the justified indicator, subject 2 (S2) was able to express his knowledge based on his own opinion, in detail, and could prove his knowledge based on the literature. Subject S2 accurately wrote down the changes that occurred, but there were some inaccuracies in the transformation process. Figure 4 displays the results of subject S2's completion, and Table 4 displays the interview transcript for the systematic indicator.

Question:
Observe points A and A'. What changes occur? Explain and write down the form of the transformation. $A = \binom{1}{2}, A' = \binom{2}{1}$.



A reflection occurs about the line $y = x$, where the initial coordinate A is at point (1, 2), then changes to A' (2, 1).

Question:
Observe points A' and A". What changes occur? Explain and write down the form of the transformation. $A' = \binom{2}{1}, A'' = \binom{-1}{2}$.



A rotation occurs through the 90° axis with the point (0, 0), where the initial coordinate A' is at point (2, 1), then changes to (-1, 2).

Figure 4. Results of completing the S2 subject on justified and systematic indicator questions

In the systematic indicator, subject 2 (S2) was able to explain a concept systematically using structured and logical sentences. The following is an excerpt from an interview with subject S2.

R: "Which are the starting point, the shadow point, and the transformation matrix?"
S1: The starting point is at coordinates $(1,2)$*, and the shadow point is* $(2,1)$*. The transformation matrix is the transformation matrix about the* $y = x$ *axis, which is* $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$.
R: "Is that written correctly? How do you know it's correct or not?"
S1: I think it's written correctly. First, write the shadow point, which is (x', y')*, followed by the matrix and the starting point. I know it's correct because I followed the text.*
R: "How do you ensure that the change from A to A' is a reflection about $y = x$*, and the change from A' to A'' is a 90° rotation with the center* $(0,0)$*?"*
S1: To confirm this, you can use the GeoGebra application to check whether it's correct or not.

On the accuracy indicator, subject 3 (S3) was able to explain his knowledge in his own words, according to the facts, precisely. Subject S3 wrote down the exact meaning of the composition of geometric transformations using his own language. The results of subject S3's completion on the accuracy indicator can be seen in Figure 5 below.

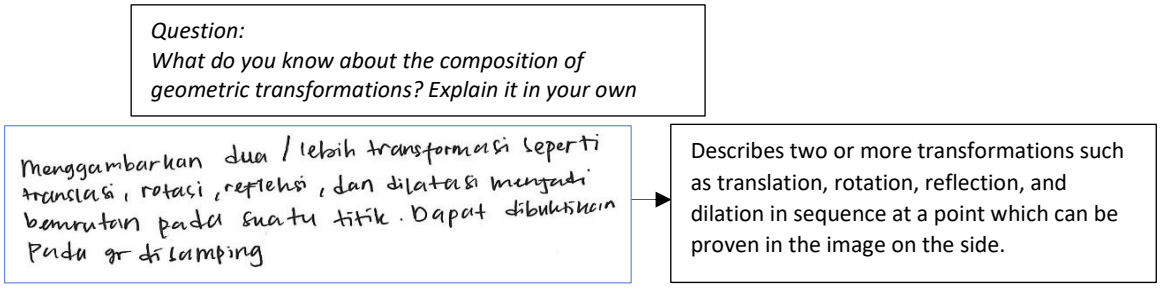


Figure 5. Results of completing the S3 subject on accurate indicator questions

In the justified indicator, subject 3 (S3) is able to express knowledge based on their own opinions, in detail, and can prove their knowledge based on the literature. Subject S3 writes down the changes that occur accurately and is able to express knowledge about geometric transformations based on their knowledge in accordance with the material that has been taught previously.

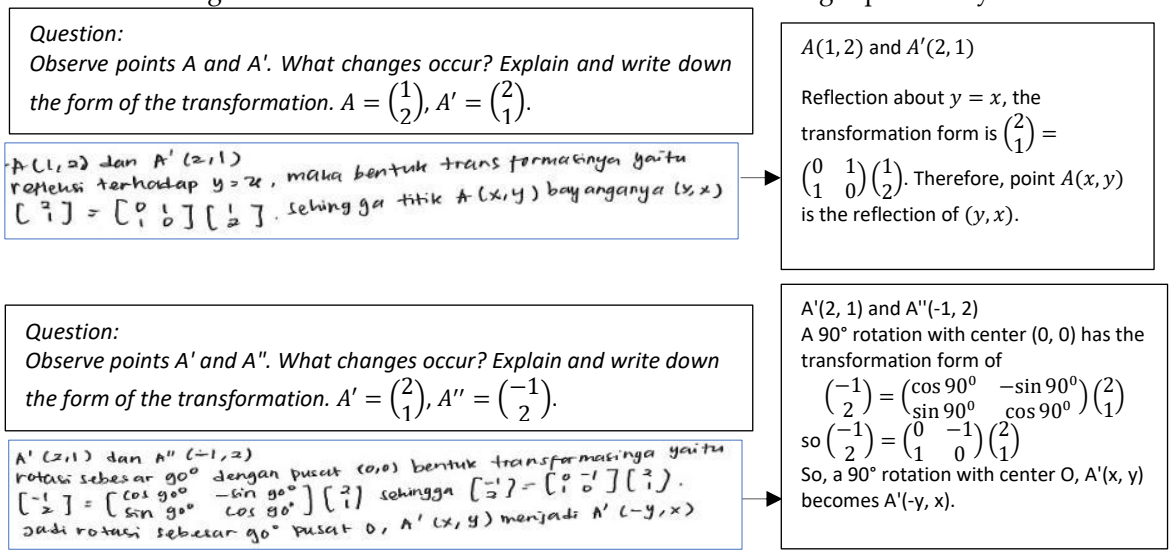


Figure 6. Results of completing the S3 subject on justified and systematic indicator questions

In the systematic indicator, subject 3 (S3) was able to explain concepts systematically using structured and logical sentences. Based on the results of the work and interviews, S3 was able to explain the concept of geometric transformation well. The subject demonstrated the ability to explain the steps of transformation, such as reflection on the line y = x and rotation by 90° with the center at (0,0), in a structured manner. In addition, S3 used logical arguments and supported his explanations with predicted results and references to geometric transformation concepts. The following is an excerpt from the interview with subject S3.

R: "Which are the starting point, the image point, and the transformation matrix?"

S1: The starting point is (1,2), the image point is (2,1), and the transformation matrix is $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

R: "Is that written correctly? How do you know it's correct?"

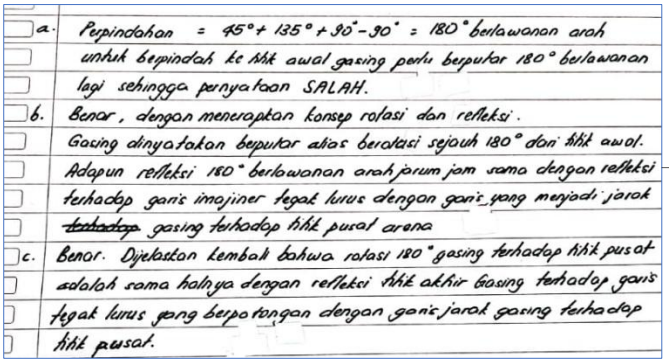
S1: I think it's correct. The image is placed on the left, and the starting point and transformation matrix are on the right.

R: "How do you ensure that the transformation from A to A' is a reflection about y=x, and the transformation from A' to A'' is a 90° rotation with the center (0,0)?"

S1: "The method is to try whether the reflection on $y = x$ *will produce the end point (y, x). What matrix, if multiplied by the starting point, will produce the shadow point, namely (y, x). For the change that occurs from A 'to A'' is a rotation of 90° with the center (0,0), well yesterday it was known that the rotation transformation uses the matrix* $\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$*. Well from there we can see which cos will produce 0, and also it is still in quadrant 1 so the results of sin and cos are all positive, from the known shadow point the numbers do not change so it can be concluded that the center is (0,0)."*

Application Aspects

In the effective indicator, subject 1 (S1) is able to use the knowledge or skills that have been learned. Subject S1 is able to apply the concept of geometric transformation to solve contextual problems given according to the knowledge that has been learned, subject S1 is also able to explain the reasons for the chosen answer choices. The results of subject S1's completion of the effective indicator can be seen in Figure 7 below.



a. Displacement = 45^0 + 135^0 + 90^0 - 90^0 = 180^0 counterclockwise. To move from the starting point, the top needs to rotate 180^0 counterclockwise, so the statement is false.
b. True, by applying the concepts of rotation and reflection. The top is said to rotate 180° from the starting point. The 180° counterclockwise reflection is the same as reflecting against an imaginary line perpendicular to the line that represents the distance of the top from the center of the arena.
c. True. It is explained again that the 180° rotation of the top about the center point is the same as reflecting the end point of the top against a perpendicular line that intersects the line of distance of the top from the center point.

Figure 7. Answers to the S1 subject application test on the effective indicator

` Subject 1 (S1) did not demonstrate good mastery of the fluency indicator, resulting in a lack of smooth implementation and several errors. While the answer choices were correct, the reasoning provided by subject S1 was inaccurate, and there were errors in the trigonometric results written in matrix form. Subject S1's fluency indicator results can be seen in Figure 8 below.

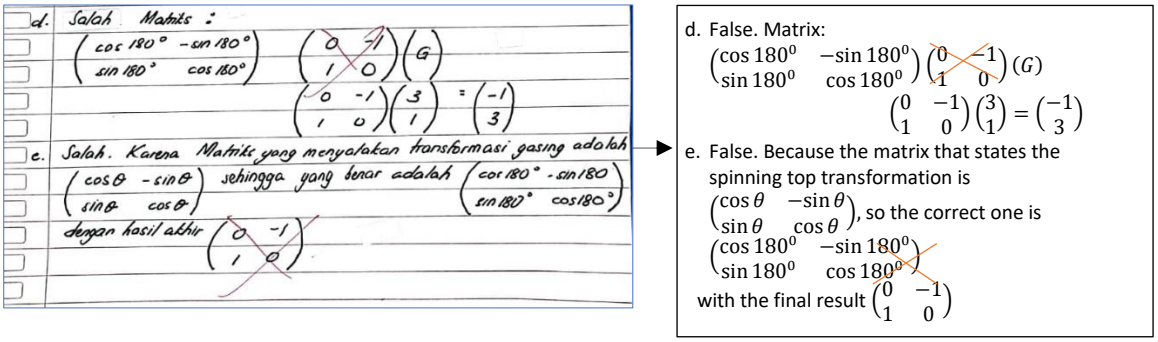


Figure 8. Answers to the S1 subject application test on the fluent indicator

In the effective indicator, subject 2 (S2) is able to achieve the desired results by using the knowledge or skills that have been learned. Subject S2 is able to apply the concept of geometric transformation to solve the given contextual problem according to the knowledge that has been learned, subject S2 is also able to explain the reasons for the chosen answer choices. The results of subject S2's completion of the effective indicator can be seen in the following figure.

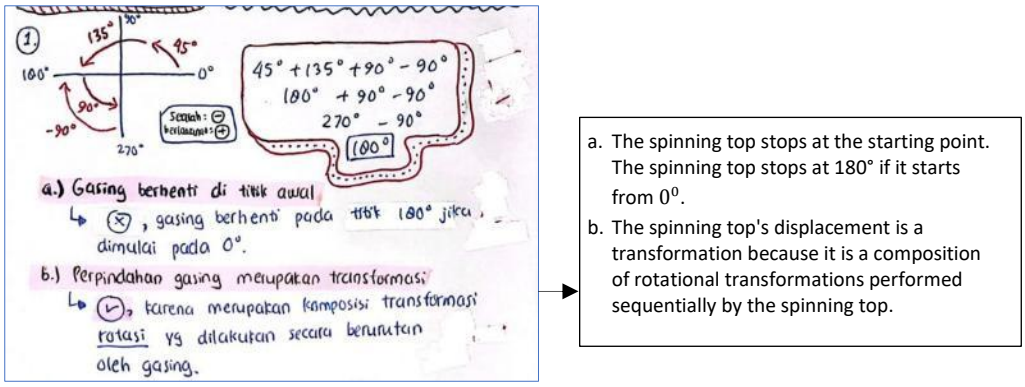


Figure 9. Answers to the S2 subject application test on the effective indicator

Subject 2 (S2) did not demonstrate good mastery of the fluency indicator, resulting in a lack of fluency and several errors. In part (d), the answer and the process were correct, but the final result contained an error in the matrix product. In part (e), the reasoning given was incorrect. The matrix form of the composition was also incorrect. Subject S2's results for the fluency indicator can be seen in the following figure.

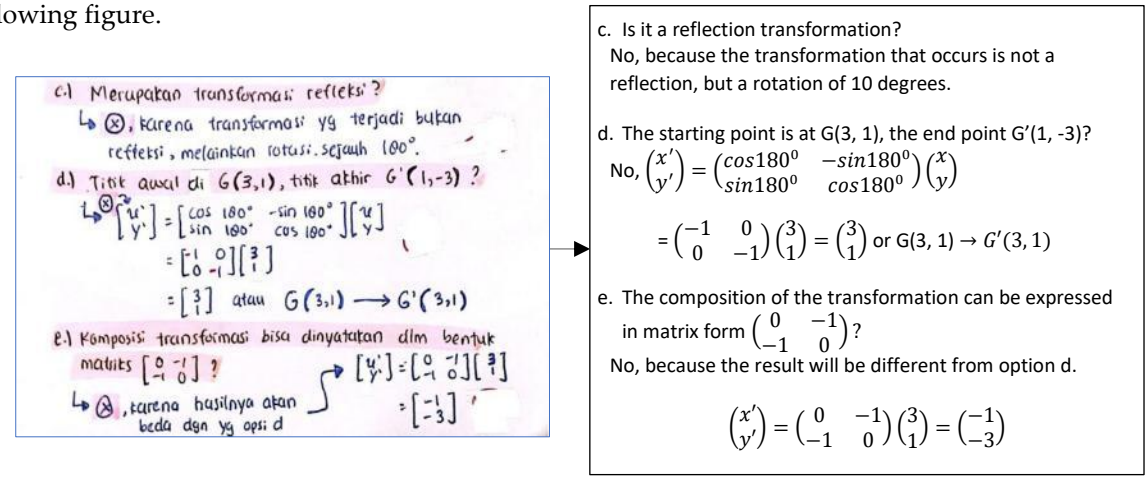


Figure 10. Answers to the S2 subject application test on the fluent indicator

In the effective indicator, subject 3 (S3) is able to achieve the desired results by using the knowledge or skills that have been learned. Subject S3 is able to apply the concept of geometric transformation to solve the given contextual problems according to the knowledge that has been learned, subject S3 is also able to explain the reasons for the chosen answer choices. In the fluent indicator, subject 3's readiness to learn develops (S3) has shown good mastery so that the application runs smoothly and without many errors. The reasons given are appropriate and the process to obtain the matrix form is correct. The results of subject S2's completion on the effective and fluent indicators can be seen in the following figure.

Given: Displacement:

T_1: counterclockwise 45^0

T_2: counterclockwise 135^0

T_3: counterclockwise 90^0

T_4: clockwise 90^0

Total displacement = 45+ 135+ 90- 90 = 180

a. False, Because the total rotation is averaged out and does not return to the starting point or does not return to point 0 again.
b. True, Because the top rotates four times, meaning it rotates four times sequentially, it rotates four times. This corresponds to the definition of function composition, which is the combination of two or more transformations.
c. Since the top rotates 180°, its matrix is $G=\begin{pmatrix}\cos 180 & -\sin 180\\ \sin 180 & \cos 180\end{pmatrix}=\begin{pmatrix}-1 & 0\\ 0 & -1\end{pmatrix}$, so its reflection is a reflection about the center point.
d. True. $\begin{pmatrix}x'\\ y'\end{pmatrix}=(x',y')=T_4{\circ}T_3{\circ}T_2{\circ}T_1$
$=\begin{pmatrix}\cos 180 & -\sin 180\\ \sin 180 & \cos 180\end{pmatrix}\begin{pmatrix}x\\ y\end{pmatrix}=\begin{pmatrix}-1 & 0\\ 0 & -1\end{pmatrix}\begin{pmatrix}3\\ 1\end{pmatrix}$
$\begin{pmatrix}x'\\ y'\end{pmatrix}=\begin{pmatrix}-3\\ -1\end{pmatrix}$, *so end point is* $(x',y')=(-3,1)$.
e. The rotation is 180^0. So can noted with this matrix:
$\begin{pmatrix}\cos 180 & -\sin 180\\ \sin 180 & \cos 180\end{pmatrix}\begin{pmatrix}-1 & 0\\ 0 & -1\end{pmatrix}$. So, this is false.

Figure 11. S1 subject application test answers on effective and fluent indicators

Based on the description above, Subject 1 (S1) met all indicators in the explanation aspect: Accurate, Justified, and Systematic. In the application aspect, Subject S1 met only one indicator: Effective. Subject 2 (S2) met all indicators in the explanation aspect: Accurate, Justified, and Systematic. In the application aspect, Subject S2 met only one indicator: Effective. Subject 3 (S3) met all indicators in the explanation aspect: Accurate, Justified, and Systematic. In the application aspect, Subject S3 met all indicators: Effective and Fluent.

The results of the study show that all three subjects (S1, S2, and S3) consistently achieved the explanation aspect, fulfilling all indicators: accurate, justified, and systematic. These findings indicate that conceptually, all three subjects had a good understanding of the material they were studying. Their ability to explain accurately and systematically demonstrates a relatively well-organized knowledge structure, while the justification indicator confirms that they were not simply providing answers but also providing logically supporting reasons.

However, differences began to emerge in the application aspect. S1 and S2 only met the effectiveness indicator, but did not demonstrate fluency in applying the concept to the given problem. This indicates that although the resulting solution was correct or on target, the process of applying the concept was not smooth and flexible. This condition indicates a tendency toward strong conceptual mastery at the declarative level, but not yet fully developed at the procedural and contextual transfer levels. In other words, the subjects were able to explain the "what" and "why," but were not yet optimal in demonstrating "how" the concept was adaptively applied to more complex situations.

Conversely, S3 demonstrated more comprehensive performance, meeting both indicators in the application aspect: effectiveness and fluency. This indicates that S3 is not only capable of producing correct solutions but can also apply the concept fluently, structured, and without significant obstacles. This fluency reflects a more mature level of integration of conceptual and procedural knowledge, thus enabling more optimal knowledge transfer.

Differences in achievement in the application aspect reinforce that good conceptual understanding does not always automatically translate to application skills Fauziyah & Hakim, 2025; Hiebert, 1986; Lenz et al., 2024). This phenomenon is often found in learning practices that emphasize the ability to explain concepts rather than using concepts in authentic contexts. If the learning process is more dominated by conceptual elaboration and structured routine practice, students tend to excel in explanation but lack the flexibility to apply them to varied situations.

These findings imply the importance of learning designs that provide more space for contextual application activities, non-routine problem solving, and reflection on solution strategies (Rajadurai & Ganapathy, 2023; Vlachopoulos & Makri, 2024). This way, students not only develop accurate and systematic understanding but also are able to transfer and apply concepts fluently across diverse situations.

Overall, the results of this study indicate a gap between the ability to explain and the ability to apply in some subjects. Therefore, pedagogical interventions that balance conceptual reinforcement with meaningful application exercises are crucial to improving the overall quality of learning.

These findings align with previous research, which found that students with moderate levels of conceptual understanding were able to understand the indicator of remembering or restating concepts, while those with less understanding of the indicator of connecting or integrating interrelated concepts were less likely to achieve the indicator of applying concepts to problems that needed to be solved or experiencing conceptual incomprehension (Armutcu & Bal, 2023; Safitri et al., 2021). This is due to the influence of the learning process, which places greater emphasis on understanding concepts through explanation and less emphasis on applying concepts to everyday life problems (Don et al., 2023; Wong, 2023).

CONCLUSION

Students' conceptual understanding of geometric transformations varied depending on their level of learning readiness. Students with developing learning readiness were able to meet all indicators in the explanation aspect, namely accurate, justified, and systematic. However, in the application aspect, only some students were able to meet the effective indicator, and only one student met the fluent indicator. The difference in ability was evident in Subject 3, who demonstrated better conceptual understanding than Subjects 1 and 2, particularly in fluent application of the concept. Challenges faced by students, such as errors in matrix calculations and interpretation of transformation results, demonstrate the importance of a more personalized learning approach.

Based on the research findings, it is recommended that other researchers further explore students' mathematical conceptual understanding by involving all learning readiness categories: advanced, developing, and needing guidance. This aims to broaden the range of research findings and provide a more comprehensive picture of students' conceptual understanding at various levels of learning readiness. Furthermore, researchers can also incorporate other variables or aspects that influence conceptual understanding, such as learning interests, learning styles, or the use of specific learning media. It is hoped that broader and more in-depth findings can be used by educators as a reference for developing more effective learning strategies, thereby improving students' overall understanding of mathematical concepts. Therefore, this follow-up research can contribute to efforts to develop more adaptive and innovative mathematics learning.

The results of this study provide recommendations for mathematics instruction, aiming to better facilitate students' problem-solving in applying mathematical concepts. The ability to apply concepts is influenced by conceptual understanding, while conceptual understanding is determined by students' learning readiness. Learning readiness refers to the readiness to understand prerequisite material. Prerequisite material is crucial because mathematics is hierarchical. Therefore, teachers can consider identifying prerequisite material skills before learning. Furthermore, they can provide treatment for students with insufficient prerequisite material skills before moving on to the core material.

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CONFLICTS OF INTEREST

The authors declare that there is no conflict of interest regarding the publication of this paper. The research was conducted in the absence of any commercial or financial relationships that could be construed as a potential conflict of interest.

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